

MATH 602 ASSIGNMENT 3, FALL 2020

**Part A.** From Gallier–Shatz:

- problem 23
- problem 24
- problem 29
- the second part of problem 30 (“entries in  $\mathbb{Q}$ ”).
- problem 34

**Part B.**

1. Find all ideals in the group ring  $\mathbb{C}[(\mathbb{Z}/4\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})]$ .
2. Let  $\mathbb{C}\langle x, y \rangle$  be the free associative  $\mathbb{C}$ -algebra in two generators  $x, y$ , isomorphic to the tensor algebra over  $\mathbb{C}$  attached to a two-dimensional vector space over  $\mathbb{C}$ . Let  $I$  be the smallest ideal of  $\mathbb{C}\langle x, y \rangle$  generated by the element  $xy - yx - 1$ , and let  $R := \mathbb{C}\langle x, y \rangle / I$ . Find a *simple* left module for  $\mathbb{C}\langle x, y \rangle$ . (You need to show that the module you find is indeed simple, i.e. it is non-zero and the only submodules are itself and  $(0)$ .)
3. (extra credit) The first part of Gallier–Shatz problem 30 (proof or counter-example).