Math 4250 Problem Set 2, Spring 2024

Part 1. From Strauss, *Partial Differential Equations*, Chapter 1.

- Exercise 1.2, #10, page 10
- Exercise 1.3, #6, page 19. [For the Laplacian in polar coordinates, see page 157 equation (5). Here you are seeking a solution that does not depend on the angle variable θ.]
- Exercise 1.5, #3, page 27
- Exercise 2.1, #2, page 38
- Exercise 2.1, #7, page 38

Part 2.

1. Let $u(x, y)$ be a smooth function on $\mathbb{R}^2$ such that

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0.$$

Show that $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}$ is constant on the line $\{x - y = c\}$ for every $c \in \mathbb{R}$, and $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is constant on the line $\{x + y = b\}$ for every $b \in \mathbb{R}$.

2. Find a solution $u(x, t)$ on the half plane $\{(x, t) \in \mathbb{R}^2 : t \geq 2\}$ of the PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x\, \partial t} - 6\frac{\partial^2 u}{\partial t^2} = 0$$

subject to the initial conditions $u(x, 0) = \phi(x)$, $\frac{\partial u}{\partial t}(x, 0) = \psi(x)$.

3. (Extra credit) Strauss, Exercise 1.3, #10, page 20.

4. (Extra credit) Strauss, Exercise 2.2, #6, page 41.