

## MATH 4100 HOMEWORK 7, SPRING 2023

Part 1. From Ash–Novinger, *Complex Variables*.

- Ch. 4, pp. 5–6, #12
- Ch. 4, pp. 11–12, #1,
- Ch. 4, pp. 11–12, #4 (a), (c),

Part 2.

(1) Evaluate the integral

$$\oint_C \frac{z}{(z^2 - 1)(z^2 + 1)} dz,$$

where  $C = \{z \in \mathbb{C} : |z - 1| = \sqrt{3}\}$ , oriented counter-clockwise.

(2) (extra credit) For every positive real numbers  $R$ , let  $C_R$  be the half-circle  $\{z \in \mathbb{C} : |z| = R, \operatorname{Im}(z) \geq 0\}$  going from  $R$  to  $-R$ . Show that

$$\lim_{R \rightarrow \infty} \int_{C_R} \frac{z e^{\sqrt{-1}z}}{z^2 + 1} = 0.$$