Math 4100 Homework 6, Spring 2023

Part 1. From Ash–Novinger, *Complex Variables*.

- Ch. 3, pp. 8-9, #4.
- Ch. 4, p. 5–6, #3. (Note: This rational function is holomorphic in many annuli centered at $z = -1$, for instance $\{0 < |z+1| < 1\}$, $\{1 < |z+1| < 3$, and $\{|z+1| > 3.\}$)
- Ch. 4, pp. 5–6, #5. Find the terms up to z^3 of the Laurent series expansion at $z = 0$ of this meromorphic function.
- Ch. 4, pp. 5–6, #10.

Part 2.

(1) (extra credit) The Fibonacci numbers are defined by

$$c_0 = 0, \quad c_1 = 1, \quad c_n = c_{n-1} + c_{n-2} \quad \text{for all} \quad n \geq 2.$$

Show that the c_n's are the Taylor coefficients at $z = 0$ of a rational function $f(z)$, and determine a closed expression for the numbers c_n.