Math 4100 Homework 3, Spring 2023

Part 1. From Ash–Novinger, *Complex Variables*.

- Ch. 2, pp. 17–19, #9
- Ch. 2, pp. 17–19, #14
- Ch. 2, p. 21, #3

Part 2.

(1) (a) Find the real and imaginary parts of $\exp(e^z)$.

(b) Express all values of $2^{\sqrt{-1}}$ and $\sqrt{-1}^{\sqrt{-1}}$.

(c) Express $\arctan w$ in terms of the logarithm.

(2) Let $D_1 = \{z \in \mathbb{C} : |z+2| + |z-2| \le 4\sqrt{2}$. $D_2 = \{z \in \mathbb{C} : 4 \le |z| \le 16\}$. Compute the integrals

$$\int_{\partial D_1} \frac{dz}{z^2+1}$$

and

$$\int_{\partial D_2} \frac{dz}{z^2+1}.$$

(3) (extra credit) Start with the definition

$$\exp(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

of the exponential function. *Prove rigorously* that there exists a real number $a \neq 0$ such that $\exp(z+a) = \exp(z)$ for all $z \in \mathbb{C}$, and every complex number b with property that $\exp(z+b) = \exp(z)$ for all $z \in \mathbb{C}$ is an integer multiple of a.

[Of course the number a above is either 2π or -2π. The above statement is a "familiar fact" you are used to. But that's not the same as an actual proof. Note that you cannot just say that cos and sin are periodic with period 2π, and that $\exp z = \cos z + \sqrt{-1}\sin z$. Both cos and sin are defined by power series, and can be expressed in terms of the exponential function. So showing that cos and sin are periodic with period 2π is equivalent to showing that exp is periodic with period 2π.]