Math 4100 Homework 1, Spring 2023

Part 1. From Ash–Novinger, *Complex Variables*, Chapter 1, pp. 9–10

- #6
- #12
- #16

Part 2.

(1) Describe geometrically the sets of points determined by the relations:

  (a) $|z+i| < 5$;
  (b) $2 > Re\{z\} > -3$;
  (c) $(Im\{z\})^2 \leq Re\{z\}$;
  (d) $|2i - z| = |z + 1 + 3i|$;
  (e) $Re\{(3-4i)z\} < 0$;
  (f) $2Re\{z\} < |z|^2$.

(2) The trigonometric functions $\cos z, \sin z : \mathbb{C} \to \mathbb{C}$ are defined by

$$\cos z = \frac{e^{\sqrt{-1}z} + e^{-\sqrt{-1}z}}{2}, \quad \sin z = \frac{e^{\sqrt{-1}z} - e^{-\sqrt{-1}z}}{2\sqrt{-1}}.$$

The functions $\tan z, \cot z, \sec z,\ \csc z$ are defined in terms of $\cos z$ and $\sin z$ as usual.

  (a) Find the values of $\sin\sqrt{-1}$, $\cos\sqrt{-1}$ and $\tan(1+\sqrt{-1})$.
  (b) Use the addition formula to separate $\cos(x+\sqrt{-1}y)$ and $\sin(x+\sqrt{-1}y)$ into real and imaginary parts, where x, y are real numbers.
  (c) Determine the subset of $\mathbb{C}$ consisting of all complex numbers z such that $\sin z = 0$.

(3) (extra credit) Let n be a positive integer, and let $a \neq 0$ be a complex number.

  (a) Show that there are exactly n distinct complex numbers $z_1, \ldots, z_n$ such that $z_i^n = a$ for $i = 1, \ldots, n$.
  (b) Show that $z_1 + \ldots + z_n = 0$. (Hint: Treat first the case when $a = 1$.)