Math 350 Assignment 5.

The "show-off problems" are starred. You are encouraged to come to my office and show me your solution. I will keep a record of people who are the first to solve any given starred problem.

Part I. Problems from Rosen's book.

- 12.2, #6 (p. 476)
- 12.3, #4 (p. 488), *#9 (p. 489), #12 (p. 489), *#14 (p. 489)
- 12.4, #2 (b) (p. 502), #8 (p. 503), *#18 (p. 504), *#19 (p. 504)

Part II.

*1. Let $x_0, x_1, \ldots, x_n, \ldots$ be variables. For every $n \geq 0$, denote by $A_n = (a_{ij}))$ the $(n+1) \times (n+1)$ matrix with diagonal entries $a_{ii} = x_i$ for $i = 0, 1, \ldots, n$, $a_{i,i+1} = -1$ for $i = 0, 1, \ldots, n-1$, $a_{i+1,i} = 1$ for $i = 0, 1, \ldots, n-1$, and all other entries are 0. (Notice that the only non-zero entries of A_n are those on the diagonal, and those immediately above or below the diagonal.) Let $P_n(x_0, \ldots, x_n) = \det(A_n) \in \mathbb{Z}[x_0, x_1, \ldots, x_n]$, a polynomial with coefficients in $\mathbb{Z}$. Prove that

$$[a_0; a_1, \ldots, a_n] = \frac{P_n(a_0, \ldots, a_n)}{P_{n-1}(a_1, \ldots, a_n)}$$

for all integers $a_0 \in \mathbb{Z}$, $a_1, \ldots, a_n \geq 1$.
(In the notation used in the lecture, we have $P_n(a_0, a_1, \ldots, a_n) = \langle a_0, a_1, \ldots, a_n \rangle = p_n$.)