Math 350 Assignment 1

The "showoff problems" are starred. You are encouraged to come to my office and show me your solution. I will keep a record of people who are the first to solve any given starred problem.

Part I.. Problems from Rosen's book.

- 2.3, #16, #20, #22* (pp. 65–66)
- 3.2, #22* (p. 87), #26 (p. 88)
- 3.4, #19 (p. 107), #22* (p. 107)
- 3.5, #45 (p. 119), #88 (p. 122)
- 3.7, #19* (p. 139)
- 4.1, #35* (p. 152), #40 (p. 152)
- 4.2, #17 (p. 157)
- 4.3, #13 (p. 164), #25* (p. 164), #32* (p. 167)
- 4.4, #9 (p. 174), #13* (p. 174)

Part II.
1.* Let $m \geq 2$ be a natural number. We say that $(\mathbb{Z}/m\mathbb{Z})^\times$ is *cyclic* if there exists an integer a, $(a, m) = 1$, such that for every integer b with $(b, m) = 1$, there exists an integer n such that $a^n \equiv b \pmod{m}$. (An equivalent condition is that the order of $a \pmod{m}$ is $\phi(m)$.

(a) Show that $(\mathbb{Z}/2^e\mathbb{Z})^\times$ is not cyclic if $e \geq 3$.

(b) Prove that $(\mathbb{Z}/p^e\mathbb{Z})^\times$ is cyclic for every $e \geq 1$ if p is an odd prime number. [We have given sufficient hint for this problem in class.]

(c) Find a necesary and sufficient condition for $(\mathbb{Z}/m\mathbb{Z})^\times$ to be cyclic in terms of the primary factorization of m.

2. Let p be an odd prime number. Determine the number of elements in $\mathbb{Z}/p\mathbb{Z}$ which is the square of an element of $\mathbb{Z}/p\mathbb{Z}$. Also, determine the number of elements in $\mathbb{Z}/p\mathbb{Z}$ which is the cube of an element of $\mathbb{Z}/p\mathbb{Z}$.