MATH 240 ASSIGNMENT 7, SPRING 2018

Due in class on Friday, March 16

Part 1. Read DELA 8.3–8.7.
Part 2. Problems from old final exams.

- fall 2016 final exam, problem 12

- fall 2015 final exam, problem 7

- fall 2012 makeup final exam, problem 9

- fall 2015 final exam, problem 13

- fall 2014 final exam, problems 7

- fall 2014 final exam, problem 8

- fall 2014 final exam, problem 9

- fall 2014 final exam, problem 10

- spring 2014 final exam, problem VIII

- fall 2013 final exam, problem 3

- fall 2013 final exam, problem 4

- fall 2012 final exam, problem 9

Part 3. (extra credit problem) Let V be the $\mathbb{C}$-vector space consisting of all smooth $\mathbb{C}$-valued functions on the real line $\mathbb{R}$. For every smooth fuction $f(x)$ on $\mathbb{R}$, let $M_f : V \to V$ be the linear operator "multiplication by $f(x)$" on V, i.e.

$$M_f : g \mapsto f \cdot g \quad \text{for all } g \in V$$

where $f \cdot g$ is the function $x \mapsto f(x) \cdot g(x)$ on $\mathbb{R}$. For every positive integer m and every $a \in \mathbb{C}$, show that

$$\left(\frac{d}{dx} - a \right)^m \circ M_{f(x)} = \sum_{i=0}^{m} \binom{m}{i} M_{f^{(i)}} \circ \left(\frac{d}{dx} - a \right)^{m-i},$$

where $f^{(i)}$ denotes the i-th derivative of f.