MATH 240 ASSIGNMENT 5, SPRING 2018

Due in class on Friday, February 23

Part 1. Read DELA 7.6 and 9.8. You might also want to consult the notes on how to compute matrix exponentials, which was explained on Friday 2/16 in class.

Note: You are advised to omit/ignore passages in DELA §9.8 involving differential equations, such as the concepts "fundamental matrix" and "transition matrix". We will get to these when we explain systems of linear first order ODE. Examples 9.8.3 and 9.8.4 illustrates calculation of $\exp(tA)$ for 2×2 and 3×3 matrices respectively.

Part 2.

(2a) Compute $\exp(tA)$ for the matrix A in problem 9 of §9.8 of DELA.

(2b) Compute $\exp(tA)$ for the matrix A in problem 10 of §9.8 of DELA.

Part 3. Let A be the following $5 \times 5 matrix$

$$A = \begin{bmatrix} -1 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

(i) Compute the characteristic polynomial of A.

(ii) For each of the eigenvalues of A, determine the number of Jordan blocks associated to that eigenvalue.

(Recall that the number of Jordan blocks associated to a given eigenvalue λ is at least 1 and at most equal to the multiplicity of λ as a root of the characteristic polynomial, and the sum of the sizes of the Jordan blocks associated to λ is equal to the multiplicity of λ in the characteristic polynomial.)

(iii) Compute $\exp(tA)$.

(iv) (extra credit) Find an invertible matrix C such that the $C^{-1} \cdot A \cdot C$ is in Jordan form.

Part 4. Problems from old final exams. (These problems are formulated as finding either the general solutions of a homogeneous system of first order ODE in the form $\frac{d}{dt}\vec{x}(t) = A \cdot \vec{x}(t)$, or finding a specific solution of an equation as above satisfying some initial conditions. The former is basically the same as computing $\exp(tA)$; the latter means that you in addition to finding $\exp(tA)$, you are asked to find a specific vector $\vec{c} \in \mathbb{C}^n$ so that the solution $\exp(tA) \cdot \vec{c}$ satisfies some additional initial conditions given.

- fall 2013 final exam, problem 8
- spring 2013 final exam, problem 9 (Change/rephrase the question to: find $\exp(tA)$.)
- fall 2016 final exam, problem 8 (Change/rephrase the question to: find $\exp(tA)$.)
- falll 2014, problem 6 (Change/rephrase the question to: find $\exp(tA)$.)
- spring 2014, problem X (Change/rephrase the question to: find $\exp(tA)$.)
- spring 2016 final exam, problem 14 (Change/rephrase the question to: find $\exp(tA)$, where A is the 2×2 matrix in the problem.