MATH 240 ASSIGNMENT 1, SPRING 2018

Due in class on Friday, January 19

Part 1. (a) Read DELA 4th ed. 2.1–2.6 and make sure you can do all core problems in these sections.
(b) Do (but do *not* hand in) the following problems in DELA:

§2.1 T/F review c, d, f, g, h.

§2.2 T/F review c, e, f, g, h; Problems 22, 26, 36. (Note that the *commutator* $[A,B]$ of two square matrices A,B is defined right before problem 19.)

§2.3 T/F review 4, 5, 6

§2.3 Problems 15, 17

§2.4 T/F review d, g

§2.5 T/F review b, c, d

§2.6 T/F review b, c, d, e

Part 2. Do and write up the following problems from DELA:

§2.2 Problem 21

§2.3 Problem 11

§2.4 T/F review e, h (You need to fully justify your answer.)

§2.4 Problems 18

§2.5 Problems 26

§2.6 T/F review g (You need to fully justify your answer.)

Part 3. Problems from old final exams. (Short answers are available for many old final exam problems, but you need to provide all details in your write-up.)

- Fall 2016 final exam, problem 1.
- Spring 2014 final exam, problem II.

Part 4. Questions about the structure of matrix multiplications.

1. Let A be a 4×3 matrix.

   (a) Find a matrix P such that $P \cdot A$ is the result of applying operation P_{23} to A (i.e. interchange the second and the third rows of A.)

   (b) Find a matrix M such that $M \cdot A$ is the result of applying operation $M_2(-2)$ to A (i.e. multiply the second row of A by -2.)

   (c) Find a matrix E such that $E \cdot A$ is the result of applying operation $A_{1,3}(5)$ to A (i.e. add 5 times the first row of A to the third row of A).

(d) Find a matrix Q such that $A \cdot Q$ is the cyclic permutation of the four columns of A: the first column of $A \cdot Q$ is the second column of A, the second column of $A \cdot Q$ is the third column of A, etc.

2. Let B be a 3×3 matrix. Suppose that there are non-zero vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$, each with 3 entries, such that $B \cdot \vec{v}_1 = 3\vec{v}_1$, $B \cdot \vec{v}_2 = 5\vec{v}_2$, and $B \cdot \vec{v}_3 = 3\vec{v}_3$. Let C be the 3×3 matrix with $\vec{v}_1, \vec{v}_2, \vec{v}_3$ as its three columns. Assume that C is invertible, i.e. there exists a 3×3 matrix C^{-1} such that $C \cdot C^{-1} = C^{-1} \cdot C = \mathrm{Id}_3$. Find the matrix $C^{-1} \cdot B \cdot C$.

Part 5. Extra credit problems

(i) (= problem 11 of the "challenge problem set" in the Math 240 course home page.) Consider the system of equations

$$x + y - z = a$$
$$x - y + 2z = b.$$

(a) Find the general solution of the homogeneous equation.

(b) A particular solution of the inhomogeneous equations when $a = 1$ and $b = 2$ is $x = 1, y = 1, z = 1$. Find the most general solution of the inhomogeneous equations.

(c) Find some particular solution of the inhomogeneous equations when $a = 1$ and $b = 2$.

(d) Find some particular solution of the inhomogeneous equations when $a = 3$ and $b = 6$.

[Remark: After you have done part a), it is possible immediately to write the solutions to the remaining parts.]

(ii) Let θ be an angle (in radians). Find a 2×2 matrix $R(\theta)$ such that $R(\theta) \cdot \vec{x}$ is the counterclockwise rotation of $\vec{x}$ by angle θ, for every vector $\vec{x}$ in the plane $\mathbb{R}^2$.