SYLLABUS FOR MATH 465 / MATH 501
INTRODUCTION TO GEOMETRY

Basic Syllabus

Course Description

This course is a bridge between vector calculus and differential geometry, the intrinsic mathematics of curved spaces. The course plan is to move from a study of extrinsic geometry (curves and surfaces in n-space) to the intrinsic geometry of manifolds. After the Theorem Egregium and a section on tensor algebra, we study manifolds and intrinsic geometry, namely metrics, connections, and the Riemann curvature tensor. Time permitting, we will study Euler characteristics, symmetry, homogeneous spaces, and/or applications such as General Relativity.

Math 465 is the basic course. Students taking the 501 version of the class will explore aspects of geometry more fully with a larger selection of homework problems.

Prerequisites (for undergraduates)

— Vector calculus (Math 240 or equivalent)
— Linear Algebra (Math 312 or equivalent)

Textbook

— Differential Geometry of Manifolds, S. Lovett. (Required)
— Elementary Differential Geometry, Revised 2nd Edition, B. O'Neill. (Recommended)

Topics

— Curves and surfaces in $\mathbb{R}^n$.
— Eulerian and Gaussian Curvature. The Theorema Egregium.
— Manifolds
— Tensor algebra: vectors and co-vectors, the exterior algebra, exterior and interior derivatives.
— Metrics. Distance, area, volume.
— Examples: flat, spherical, and hyperbolic spaces. Groups $SO(3)$, $SU(2)$, etc.
— Intrinsic (covariant) differentiation. The Riemann tensor.
— Possible additional topics: Euler characteristic, homogeneous spaces, Lie groups, Electromagnetism, General Relativity.

Date: *October 30, 2013.*

Detailed Syllabus

Extrinsic geometry.

1) Vector calculus review. Parametrized curves and surfaces in n-space. Unit tangent and principle normal. Geodesic curvature.
2) Surfaces: Coordinate Patches. Example: Stereographic projection. Functions and vector fields defined on patches. (O'Neill Ch. 4.1-4.4, Lovett Ch. 2)
3) Curvature: Euler and Gaussian curvature (O'Neill Ch 5.1-5.3). The Theorem Egregium.
4) Manifolds: Definition, examples of the torus and $\mathbb{S}^2$. Tangent spaces and differentials (Lovett Ch 3.1-3.3).

Tensor algebra and tensor calculus.

5) Vectors and covectors. Duality.
6) Tensor algebra: tensor products, and the exterior algebra
7) Vector bundles on manifolds. Vector fields and tensor fields on manifolds. (Lovett Ch 4.1-4.2)
8) Tensor calculus. Exterior and interior derivatives. Integration on manifolds. (Lovett Ch 4.3-4.4). Stokes theorem (optional, Lovett Ch 4.5).

Intrinsic geometry

9) Metrics. Examples of $\mathbb{S}^2$ and $\mathbb{H}^2$. Conformal and non-conformal changes of metric.
10) Covariant differentiation. Koszul formula.
11) Riemannian curvature and the Riemann curvature tensor.
12) Examples: $\mathbb{S}^2$, $\mathbb{S}^3$, and $\mathbb{H}^2$. Euler angles and the geometry of $SO(3)$.

Possible additional topics, time allowing

— Basic topology
— Euler characteristic (Lovett A.6, O'Neill 6.8)
— Electromagnetism (Lovett Ch 6.2)
— General Relativity (Lovett Ch 6.4)
— Lie groups and/or homogeneous spaces, symmetric spaces
— Symmetries and Killing fields. The Hopf fibration.