Homework 5 Worksheet

Remember, no credit will be given for answers without justification.

1 Linear Algebra

For questions (1)-(3), perform the following:

a) Determine all eigenvalues, eigenvectors, and generalized eigenvectors.

b) Indicate the multiplicity of all eigenvalues.

c) Indicate which generalized eigenvectors form chains.

d) If $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A, verify that $Tr\, A = \sum_{i=1}^{n} \lambda_i$ and that $det\, A = \prod_{i=1}^{n} \lambda_i$.

1) Let A be the 2×2 matrix

$$A = \begin{pmatrix} -\frac{5}{3} & \frac{4}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \tag{1}$$

2) Let A be the 3×3 matrix

$$A = \begin{pmatrix} -3 & -1 & -1 \\ 2 & 0 & 1 \\ -1 & -1 & -2 \end{pmatrix} \tag{2}$$

3) Let A be the 4×4 matrix

$$A = \begin{pmatrix} -4 & 1 & 1 & -2 \\ 0 & -3 & 2 & -2 \\ 1 & -1 & -2 & 0 \\ 1 & -1 & -1 & -1 \end{pmatrix} \tag{3}$$

(Yes you do have to show your work, but do NOT spend too much time on this one! Come to my office; I'll show you some tricks.)

2 Systems of Differential Equations

In questions

4) Find the general solution of the system

$$\frac{d\vec{X}}{dt} = A\vec{X} \tag{4}$$

where A is the matrix in (1). In addition, sketch the phase portrait.

5) Find the general solution of the system

$$\frac{d\vec{X}}{dt} = A\vec{X} \tag{5}$$

where A is the matrix in (2).

6) Find the general solution of the system

$$\frac{d\vec{X}}{dt} = A\vec{X} \tag{6}$$

where A is the matrix in (3).

3 Second Order Differential Equations

Recall the "standard form" for linear, homogeneous, constant-coefficient second order differential equations:

$$\ddot{y} + 2\zeta\dot{y} + \omega_0^2 y = 0. \tag{7}$$

In each of the following systems

a) State whether the system is underdamped, critically damped, or overdamped.

b) Identify the undamped natural frequency.

c) If the system is underdamped, find the system's natural frequency.

d) Write down the general solution.

7) $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0.$

8) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0.$

9) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 0.$

10) $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 9y = 0.$