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Afterword (1994)

Each reprinting of this text has given me an opportunity to correct misprints and errors pointed out by patient readers. The most substantial change was the addition of the appendix to §24 at the time of the second printing. If I were starting today from scratch, I would certainly do some things—large and small—differently. In the area of notation, for example, ρ is now more commonly used than δ to denote the half-sum of positive roots. But with many copies of the book already in circulation, I have been reluctant to disturb the existing format.

While the structure theory developed in Chapters I–V has evolved very little over the past 25 years, there has been an explosion of new work in representation theory. The foundations laid here in Chapter VI are still valid, being aimed primarily at the classical finite dimensional theory of Cartan and Weyl. However, some of the *ad hoc* terminology and notation I introduced have long since been replaced by other conventions in most of the literature: In place of "standard cyclic modules" one now speaks of "highest weight modules", the universal ones being called "Verma modules". Verma modules are usually denoted $M(\lambda)$ rather than $Z(\lambda)$, while the irreducible quotient is labelled $L(\lambda)$. Of course, Lie theory is a sprawling subject, with many conflicting notational schemes (especially for root systems). So the student has to be somewhat flexible in any case when approaching the literature.

The present text contains much of the standard core of semisimple Lie theory, in a purely algebraic setting. This theory—especially the classification of simple Lie algebras by Dynkin diagrams—is beautiful in its own right, whatever one's ulterior motive for studying it. But readers should be aware of the far-reaching developments of recent decades that rely in some way on this core. While it is impossible in a page or two to survey these developments adequately, a quick overview may be useful. References below are mainly to books rather than to the multitude of original articles; the latter are well documented in the annual subject index of *Mathematical Reviews*. With due apology for omissions, here are some of the subjects most closely related to semisimple Lie algebras:

- *The BGG category $\mathcal{O}$.* This consists of finitely-generated weight modules on which a fixed Borel subalgebra acts locally finitely. It includes Verma modules and irreducible highest weight modules $L(\lambda)$ for arbitrary λ, as well as projective and injective objects. Besides the BGG resolution of a finite dimensional $L(\lambda)$ by Verma modules, which makes more concrete the derivation of Weyl's character formula presented here, one encounters BGG reciprocity along with the Jantzen filtration and sum formula: see J. C. Jantzen, *Moduln mit einem höchsten Gewicht*, Lect. Notes in Mathematics 750, Springer-Verlag, 1979.

- *Kazhdan–Lusztig conjectures.* A conjectured character formula for all $L(\lambda)$ appeared in the seminal paper by D. A. Kazhdan and G. Lusztig, Representations of Coxeter groups and Hecke algebras, *Invent. Math.* **53** (1979), 165–184. This formula was quickly proved (independently) by Beilinson–Bernstein and by Brylinski–Kashiwara, using a dazzling array of techniques. The Hecke algebra approach has become extremely influential in several kinds of representation theory.
- *Primitive ideals in enveloping algebras.* Combining noncommutative ring theory and algebraic geometry with the representation theory of semisimple Lie algebras yields deep results on the structure of universal enveloping algebras. See J. Dixmier, *Algèbres enveloppantes*, Gauthier-Villars, 1974, and J. C. Jantzen, *Einhüllende Algebren halbeinfacher Lie-Algebren*, Springer-Verlag, 1983.
- *Lie group representations.* Lie algebra techniques indicated above have led to decisive progress in many areas of the representation theory of semisimple (or reductive) Lie groups. See for example D. A. Vogan, Jr., *Representations of Real Reductive Lie Groups*, Birkhäuser, 1981.
- *Representations of algebraic groups.* Much of the theory of semisimple Lie algebras can be adapted to semisimple algebraic groups in arbitrary characteristic. Representations in characteristic p are somewhat like infinite dimensional representations in characteristic 0. See J. C. Jantzen, *Representations of Algebraic Groups*, Academic Press, 1987.
- *Finite groups of Lie type.* Lie theory is essential to understanding the structure of these groups, as well as their ordinary and modular representations. See R. W. Carter, *Finite Groups of Lie Type: Conjugacy Classes and Complex Characters*, Wiley, 1985.
- *Kac-Moody Lie algebras and vertex operators.* The Serre relations of §18 lead to new classes of infinite dimensional Lie algebras, when the Cartan matrix is replaced by a "generalized Cartan matrix". These Kac-Moody Lie algebras and their representations interact deeply with mathematical physics, combinatorics, modular functions, etc. See V. G. Kac, *Infinite Dimensional Lie Algebras*, 3rd edition, Cambridge University Press, 1990, and I. Frenkel, J. Lepowsky, A. Meurman, *Vertex Operator Algebras and the Monster*, Academic Press, 1988.
- *Quantum groups.* Since the pioneering work of Drinfeld and Jimbo in the mid-eighties, quantized enveloping algebras have become ubiquitous in mathematics and mathematical physics. See G. Lusztig, *Introduction to Quantum Groups*, Birkhäuser, 1993, and J. Fuchs, *Affine Lie Algebras and Quantum Groups*, Cambridge University Press, 1992.
- *Combinatorics, geometry, etc.* Apart from their connection with Lie algebras, root systems and root lattices along with related Coxeter groups such as Weyl groups play an essential role in many areas: Macdonald formulas, quivers and representations of finite dimensional algebras, singularities, crystals and quasi-crystals, etc. See for example J. H. Conway, N. J. Sloane, *Sphere Packings, Lattices, and Groups*, Springer-Verlag, 1993.

Index of Terminology