Sample Math / Maple Project:

Exploring limits and plotting - First steps in Maple

In Maple, functions can be defined in one of two fairly self-explanatory ways: The first way is more-or-less standard mathematical notation:

> **`f:= x -> x^2-3*x;`**

$$f := x \rightarrow x^2 - 3\,x$$

To make the arrow in the function definition, type the minus sign followed by the greater than sign. Standard mathematical notation for the above would be f(x)=x^2-3x.

The other way is to define a "procedure". This is especially useful if the function is complicated and requires more than a simple formula to express:

> **`h:= proc(x) sin(k*x)/x end;`**

$$h := \mathbf{proc}(x)\ \sin(k{*}x)\,/\,x\ \mathbf{end}$$

In each way of defining a function, notice that the name of the independent variable (or variables, there can be more than one) is explicitly given. Notice also that the second kind of function definition must end with the word "end".

The function h has a parameter in it (the 'k') - to get numerical results from h, it is necessary first to assign a value to k.

Evaluating functions is done using standard mathematical notation:
> **`f(3);`**

$$0$$

```
> h(4);
```

$$\frac{1}{4} \sin(4\,k)$$

```
> h(0);
Error, (in h) division by zero
```

Let's give k a value, so we can do some numerical work:

```
> k:=2:
```

The colon at the end of the line (instead of a semicolon) supresses the output.

```
> h(3);
```

$$\frac{1}{3} \sin(6)$$

Since sin(6) is not an easily expressible number, Maple leaves it as it is. If you want Maple to give a decimal approximation, use evalf:

```
> evalf(h(3));
```

$$-.09313849939$$

To study functions numerically, it is easy to make tables of values using the iterative "for" construction:

```
> for x from 0.1 by 0.1 to 1.0 do print(x,f(x),h(x)) od;
```

.1, -.29, 1.986693308
.2, -.56, 1.947091712
.3, -.81, 1.882141578
.4, -1.04, 1.793390227
.5, -1.25, 1.682941970
.6, -1.44, 1.553398477
.7, -1.61, 1.407785329
.8, -1.76, 1.249467004
.9, -1.89, 1.082052923
1.0, -2.00, .9092974268

To understand functions, it is often important to study their graphs. It is easy to make graphs with Maple:

```
> plot(h(x),x=0.1..2);
```

That certainly didn't give the expected result. The reason for this is that after execution of the preceding "for" statement, the variable x had a specific value (1.0). To make x a free variable again we must "unassign" it as follows:

```
> x:='x':
> plot(h(x),x=0.1..2);
```
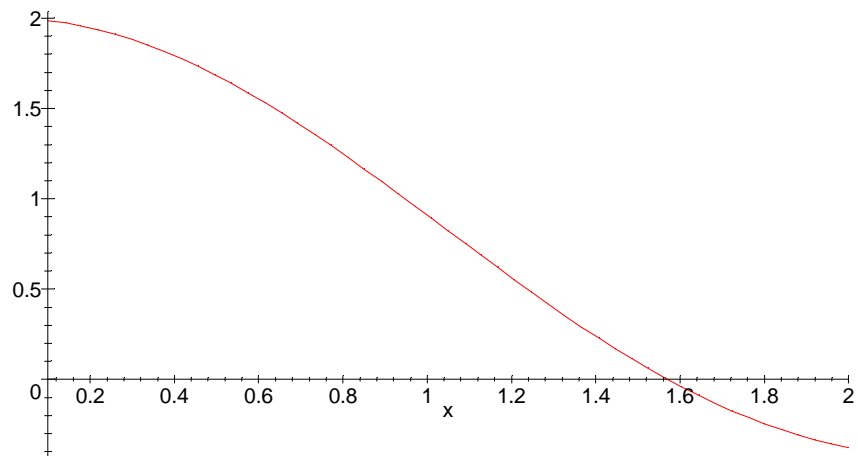

That's better!

1. Make a table of values of h(x) (with k=2) for positive values of x which approach zero. One way to do this with a "for" statement is to say something like

```
> for t from 1 to 6 do print(0.1^t,evalf(h(0.1^t))) od;
```

The limit should be pretty apparent from this.

2. Choose some (at least three) other values of k, and make similar tables. Also do a table or two with negative values of x. How does the limit of h(x) as x approaches zero depend on k? Write your answer in a complete sentence.

Limits can also be explored by plotting. Our plot above with k=2 made it pretty apparent that the limit was 2.

3. Using the same values of k as in problem 2, make plots of h(x) for x from 0.01 (say) to 2. Is it apparent from these plots how the limit depends on k?

4. Experiment with the function

```
> s:= x->(1+k*x)^(1/x);
```

$$s := x \rightarrow (1 + k\,x)^{\left(\frac{1}{x}\right)}$$

Make tables and plots to discern the limit of s(x) as x ->0 for various values of k. Is it apparent how the limit depends on k?

5. Here are a few more limits to explore using numerical and graphical analysis:

The limit as x->0 of (sec(x)-1)/x

The limit as x->Pi/2 of sec(x)-tan(x)

CHALLENGE: The limit as x->infinity of sqrt(x+5)/(sqrt(x)+5).
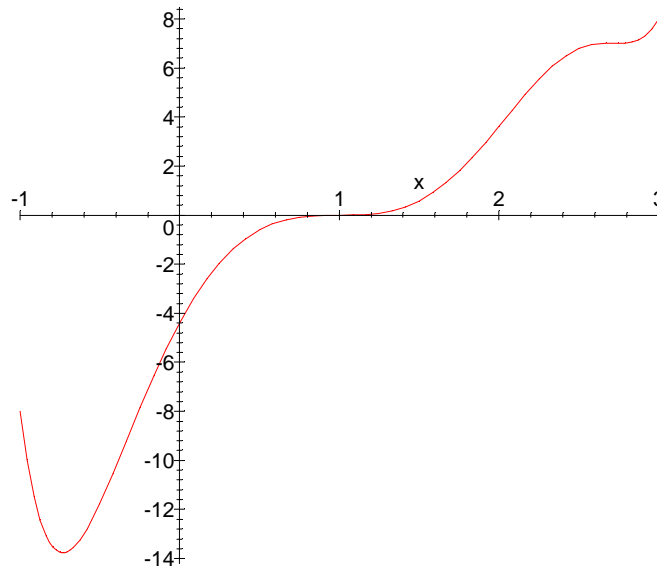The challenge here is to deal with the limit as x goes to infinity!

PLOTTING PITFALLS

It is easy to make plots with Maple - it is also easy to be fooled by them sometimes. Let's look at the graph of a polynomial of degree 8:

```
> g:= (-35/8) +12*x - 9*x^2 - 2*x^3 +(25/4)*x^4 - 5*x^5 +3*x^6 -
  x^7+x^8/8;
```

$$g := -\frac{35}{8} + 12\,x - 9\,x^2 - 2\,x^3 + \frac{25}{4}\,x^4 - 5\,x^5 + 3\,x^6 - x^7 + \frac{1}{8}\,x^8$$

```
> plot(g,x=-1..3);
```



```
>
```

It looks as though the graph of g has a low point for some x near -1, a "flex" (i.e., an inflection point) near x=1 and another flex near x=3. But it appears that the function is increasing for all x>0.

6. Examine both flex points more closely (i.e., do plots with the range of x restricted to be small intervals containing the flexes). Which one is a real flex? Which isn't? How does the function behave near the latter point? Would you have guessed this from the original plot?