Math 241 Final Exam

Name: ____________________________

Student ID: ____________________________

Signature: ____________________________

Instructions: Print your name and student ID number and sign your signature to indicate that you accept the honor code. You may use **one page** of notes on this test. You may not use any other notes, books, calculators or computers. When a box is provided for your answer you must write your answer (and nothing else) in the box to receive full credit for the problem. Even if the correct answer appears somewhere else on the page, you will not receive full credit. Moreover, you must also **show the work** you did to arrive at the answer to receive full credit.You have 2 hours to answer all the questions. *Good Luck*

Question	Max Point	Score
1	9	
2	9	
3	9	
4	5	
5	4	
6	8	
7	4	
8	5	
9	5	
10	4	
Total	62	

1) True or False: (1 point each) Circle **T** for True and **F** for False.

1. $u(x,y) = e^x \sin y$ is a solution to $u_{xx} + u_{yy} = 0$.

   T **F**

2. Let $u(x,y) = \mathrm{Im}(f(x+iy))$ where $f(z)$ is an analytic function. Then u is harmonic.

   T **F**

3. If a function $f(z)$ is analytic then $\frac{\partial f}{\partial y} = \frac{1}{i}\frac{\partial f}{\partial x}$.

   T **F**

4. If $f(x)$ and $g(x)$ are even functions then $h(x) = f(g(x))$ is odd.

   T **F**

5. If z_0 is a pole of $f(z)$ then the residue of f at z_0 is $\lim_{z \to z_0}(z - z_0)f(z)$

   T **F**

6. If u and v solve the wave equation then so does $u - 9v$.

   T **F**

7. If the Fourier series of a function converges then it converges to the value of the function at every point.

   T **F**

8. A bounded entire function is constant.

   T **F**

9. The singularity at 0 of $\tan \frac{1}{z}$ is isolated.

   T **F**

2)Short Answer: (1 point each)

1. What is the residue of $f(z) = \frac{e^z(z-1)}{z+9}$ at $z = -9$.

Answer:

2. Let $f(x) = x + 9$ on $[-\pi, \pi]$. What does the Fourier series of f converge to at π.

Answer:

3. $f(x)$ as above. What does the Fourier series of f converge to at $2 + \pi$.

Answer:

4. What is $\int_C \frac{1}{z^2} dz$ where C is $|z| = 7$ oriented counter clockwise?

Answer:

5. What are the solutions to $z^5 = 1$?

Answer:

6. Where are the singular points of $f(z) = \tan z$.

Answer:

7. What type of singularity does $f(z) = \frac{\tan z}{z}$ have at $z = 0$?

Answer:

8. Compute $\int_{-\pi}^{\pi} (\sin x + 7\cos 2x)(\sin 2x - \cos 2x)\, dx$

Answer:

9. How many different Laurent series does $f = \frac{1}{(z-1)(z^2+1)}$ have centered at $z = 1$.

Answer:

3) (3 points each) Compute the following contour integrals. All curves oriented counter clockwise.

1. $\int_C \frac{z^2-4}{z^2+1}\,dz$ where C is $|z+i|=1$.

Answer:

2. $\int_C \frac{\sin z}{(z-\pi)^2}\,dz$ where C is $|z-2|=7$.

Answer:

3. $\int_C \frac{e^z}{z\sin z}\,dz$ where C is $|z-3|=4$.

Answer:

4) 1. (3 points) Let $f(z) = \frac{e^z}{z-1}$. Find the Laurent expansion of $f(z)$ about $z = 1$ good in the region containing the point 2.5.

Answer:

2. (2 points) Find the Laurent expansion of $f(z) = \frac{1}{z-2}$ about $z = 1$ good in the region containing the point 5.

Answer:

5) (4 points) Find the Fourier series for

$$f(x) = \begin{cases} x+1 & -2 < x \leq 0 \\ x & 0 < x < 2. \end{cases}$$

Answer:

6) (4 points each) Compute

1. $\int_{-\pi}^{\pi} \frac{1}{10-6\cos\, x}\, dx$

Answer:

2. $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)}\, dx$

Answer:

7) (4 points) Compute the Fourier transform of the function

$$f(x) = \begin{cases} (3 - 6|x|) & x \in [-\frac{1}{2}, \frac{1}{2}] \\ 0 & \text{otherwise} \end{cases}$$

Answer:

8) (5 points) Suppose f, g are analytic functions in the domain D. If $\mathrm{Re}(f) = \mathrm{Re}(g)$ in D show that $f(z) = g(z) + c$ where c is some constant.
HINT: Consider $h = f - g$ and thing about the Cauchy-Riemann equations.

9) (5 points) Consider $u_{xx} + 7u_t + u = 0$ subject to the boundary conditions $u(0,t) = 0$ and $u(5,t) = 0$. Find all solutions of the form $u(x,t) = X(x)T(t)$.

Answer:

10) (4 points) Suppose $f(z) = \frac{g(z)}{h(z)}$ where g is analytic and non-zero at z_0 and $h(z)$ is analytic and has a zero of order 1 at z_0. Show

$$\mathrm{Res}(f(z), z_0) = \frac{g(z_0)}{h'(z_0)}.$$

You may use anything you know from class (except of course this formula).