Final, Math 241, Fall 2009

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You may use one sheet of 8 x 11" paper on which you write any information you like. No calculator.Good luck.
Show all work, even on multiple choice questions.

(1) Compute the principal value of the integral
$$\int_0^\infty \frac{\sin x}{x(x^2+1)}dx.$$

(a) 0
(b) $\frac{1}{2}(2-e^{-1})$
(c) $\frac{\pi}{2e}$
(d) $\frac{\pi}{2}(1-e^{-1})$
(e) $\frac{\pi}{2}(2-e^{-1})$

(2) Evaluate $\int_C \frac{\sin(2z)}{(6z-\pi)^3} dz$, where C is the ellipse given by

$$x^2 + 4y^2 = 4$$

oriented counter-clockwise.

(a) 0

(b) $1/2$

(c) πi

(d) $-\sqrt{3}$

(e) $-2\pi i\sqrt{3}$

(3) Evaluate the integral of $f(z) = z\cos(z^2)$ along the contour C that begins at 0, moves along the real axis to 1, moves counterclockwise around the circle of radius 1 until it reaches -1, then moves down along a vertical path to $-1-i$. (Hint: there is a shortcut.)

(a) 0

(b) $\frac{i}{2}(e^{-2} - e^{2})$

(c) $\frac{1}{2}(1+i)(e^{2} - e^{-2})$

(d) $\frac{i}{4}(e^{2} - e^{-2})$

(e) $\frac{1}{2}(e^{2} - e^{-2})$

(4) Compute a Laurent expansion of the function $f(z) = \frac{1}{(z-2i)(z+i)}$ valid on the annulus given by $1 < |z| < 2$.

(5) (a) Compute all possible values of $i^{\frac{\pi i}{2}}$.

(b) Compute all possible solutions of the equation $\cos(z) = 2$.

(6) Compute the eigenvalues and eigenfunctions of the Sturm Liouville problem

$$x^2y'' + xy' + 25\lambda y = 0, \text{ subject to } y'(1) = 0 \text{ and } y(e) = 0.$$

(7) Evaluate the Cauchy-Principal value of the integral

$$\int_{-\infty}^{\infty} \frac{3x^2}{(x^2+2x+2)(x^2+1)^2} dx$$

(8) For each of the following functions determine all the singularities and classify them as removable, pole (and of what order) or essential.

(a) $\frac{\cos(z)}{z^2}$

(b) $\frac{z}{\sin(z)}$

(c) $e^{1/z}/z$

(9) What is the radius of convergence of the Taylor series centered at $2+i$ of the function $\frac{\cos(z)}{z(z-\pi)}$?

(10) Suppose $u(r, \theta)$ satisfies Laplace's equations $\Delta u = 0$ on the unit disc $r \leq 1$, $\theta \in [0, 2\pi]$ with $u(1, \theta) = f(\theta)$. Calculate $u(r, \theta)$.