Math 241 Final Exam
Fall 2007

1. Find the radius of convergence for the Taylor series of $f(z) = \frac{1}{z^8 - 1}$ about the point $z = 2\sqrt{2} + 2\sqrt{2}i$.

(A) 1 (C) 2 (E) 3 (G) 4

(B) $\frac{3}{2}$ (D) $\frac{5}{2}$ (F) $\frac{7}{2}$ (H) ∞

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2. Consider the Laurent series for the function

$$f(z) = \frac{z^2 - 2z + 3}{z - 2}$$

in the region $|z-1| > 1$. What is the coefficient of the $(z-1)^{-2}$ term?

(A) -6 (C) -3 (E) 1 (G) 3
(B) -4 (D) 0 (F) 2 (H) 6

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3. Find the constant k such that the function $v(x, y) = 3x^2 y + ky^3 - x + 1$ is a harmonic conjugate of the function $u(x, y) = x^3 - 3xy^2 + y$.

(A) -3 (C) -1 (E) 1 (G) 3
(B) -2 (D) 0 (F) 2 (H) 4

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4. Evaluate

$$\int_0^{2i} e^{iz} dz.$$

(A) $i(1-e^{-2})$ (C) $1-ie^{-2}$ (E) $i-e^{-2}$ (G) $1-e^{-2}$

(B) $i(1+e^{-2})$ (D) $1+ie^{-2}$ (F) $i+e^{-2}$ (H) $1+e^{-2}$

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5. Evaluate

$$\frac{i}{2}\oint_{|z|=1} \frac{\left(z^2-1\right)^2}{z^2\left(z+\frac{1}{2}\right)(z+2)}\,dz$$

(A) $\frac{\pi}{10}$ (C) $\frac{\pi}{6}$ (E) $\frac{\pi}{3}$ (G) 1

(B) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$ (F) $\frac{\pi}{2}$ (H) π

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6. Evaluate

$$\int_0^{2\pi} \frac{d\theta}{2-\cos\theta}.$$

(A) π (C) $\frac{2\pi}{3}$ (E) $\frac{\pi}{\sqrt{3}}$ (G) $\frac{\pi}{4}$

(B) $\frac{2\pi}{\sqrt{3}}$ (D) $\frac{\pi}{2}$ (F) $\frac{\pi}{3}$ (H) $\frac{\pi}{6}$

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7. Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{x^2-6x+13}.$$

(A) $-\pi$ (C) $-\frac{\pi}{24}$ (E) $\frac{\pi}{24}$ (G) $\frac{\pi}{2}$

(B) $-\frac{\pi}{12}$ (D) 0 (F) $\frac{\pi}{12}$ (H) π

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8. In the Fourier series expansion of $f(x) = 2x^2 - 1$ on (-1,1) find the coefficient on the $\cos(4\pi x)$ term.

(A) 0 (C) $\frac{1}{2\pi}$ (E) $\frac{1}{2}$ (G) 1

(B) $\frac{1}{2\pi^2}$ (D) $\frac{2}{\pi^2}$ (F) $\frac{2}{\pi}$ (H) 2

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9. Consider the Sturm-Liouville problem defined on $0 \le x \le \frac{\pi}{2}$:

$$y'' + \lambda y = 0 \qquad y(0) = 0,\ y'\left(\frac{\pi}{2}\right) = 0 .$$

Find all eigenvalues $\lambda_n,\ n = 0,1,2,\dots$.

(A) $\lambda_n = n^2$ (C) $\lambda_n = \frac{n}{4}$ (E) $\lambda_n = \frac{(2n-1)\pi}{4}$ (G) $\lambda_n = \frac{(2n-1)^2\pi}{2}$

(B) $\lambda_n = \frac{n^2}{4}$ (D) $\lambda_n = \frac{(2n-1)\pi}{2}$ (F) $\lambda_n = (2n-1)^2$ (H) $\lambda_n = \frac{(2n-1)^2\pi}{4}$

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10. The solution $u(x,t)$ defined for $0 \le x \le 2, t \ge 0$ to the wave equation
$u_{tt} = u_{xx}$ with boundary conditions $u_x(0,t) = u_x(2,t) = 0$ is

$$u(x,t) = \sum_{n=0}^{\infty}\left[A_n \cos\left(\tfrac{n\pi}{2}t\right) + B_n \sin\left(\tfrac{n\pi}{2}t\right)\right]\cos\left(\tfrac{n\pi}{2}x\right).$$

Find $u\left(\frac{1}{3},\frac{1}{2}\right)$ with initial conditions $u(x,0) = 3\cos(\pi x)$ and $u_t(x,0) = 2\cos(3\pi x)$.

(A) $\frac{1}{\pi}$ (C) $\frac{3}{\pi}$ (E) $\frac{2}{3\pi}$ (G) $\frac{1}{2}$

(B) $\frac{2}{\pi}$ (D) $\frac{1}{3\pi}$ (F) $\frac{1}{3}$ (H) 2

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11. Let $u(x,t)$ be a function defined for $0 \le x \le \pi, t \ge 0$ such that

$$u_t = u_{xx} + 2u_x$$

with boundary conditions $u(0,t) = u(\pi,t) = 0$ for all $t \ge 0$

and initial condition $u(x,0) = e^{-x}\sin(2x)$.

Use separation of variables to find $u(\frac{\pi}{4},1)$.

(With separation constant $-\lambda$, you will find non-trivial solutions when $\lambda > 1$, say $\lambda = 1 + \alpha^2$)

(A) $e^{-1-\frac{\pi}{2}}$ (C) $e^{-5-\frac{\pi}{2}}$ (E) $e^{-1-\frac{\pi}{4}}$ (G) $e^{-5-\frac{\pi}{4}}$

(B) $e^{-2-\frac{\pi}{2}}$ (D) $e^{-10-\frac{\pi}{2}}$ (F) $e^{-2-\frac{\pi}{4}}$ (H) $e^{-10-\frac{\pi}{4}}$

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12. Consider a circular plate of radius 1 whose circular edge is maintained at the temperature $u(1,\theta) = \theta$. The steady-state temperature $u(r,\theta) = A_0 + \sum_{n=1}^{\infty} r^n \left[A_n \cos(n\theta) + B_n \sin(n\theta) \right]$ satisfies $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$. Find the coefficient of the $\sin(3\theta)$ term.

(A) 0 (C) $\frac{2}{3}$ (E) $\frac{\pi}{3}$ (G) $\frac{2\pi}{3}$

(B) $\frac{1}{3}$ (D) $\frac{-2}{3}$ (F) $\frac{-\pi}{3}$ (H) $\frac{-2\pi}{3}$

SOLUTIONS:

1. E
2. G
3. C
4. A
5. H
6. B
7. G
8. B
9. F
10. E
11. G
12. D