Math 241 Makeup Final Exam

Name: ________________________________

Student ID: ____________________________

Signature: _____________________________

Instructions: Print your name and student ID number and sign your signature to indicate that you accept the honor code. You may use **one page** of notes on this test. You may not use any other notes, books, calculators or computers. When a box is provided for your answer you must write your answer (and nothing else) in the box to receive full credit for the problem. Even if the correct answer appears somewhere else on the page, you will not receive full credit. Moreover, you must also **show the work** you did to arrive at the answer to receive full credit.You have 1.5 hours to answer all the questions. *Good Luck*

Question	Max Point	Score
1	8	
2	9	
3	9	
4	5	
5	5	
6	8	
7	5	
8	5	
9	5	
10	5	
11	4	
Total	68	

1) True or False: (1 point each) Circle **T** for True and **F** for False.

1. The function $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = \sin(iz)$ is periodic.

   T **F**

2. The partial differential equation $u_{xx} - 9u_{xy} + 7u_x + e^x u = 0$ is a parabolic differential equation.

   T **F**

3. Let $u(x, y) = \mathrm{Re}(f(x + iy))$ where $f(z)$ is an analytic function. Then u is harmonic.

   T **F**

4. A function $f(z)$ analytic in a set D may have a pole in D.

   T **F**

5. All functions have a convergent Fourier series.

   T **F**

6. The function $f(z) = |z|^2$ is entire.

   T **F**

7. The Taylor series of $f(z) = \frac{e^z}{z-5}$ about $z = 0$ has radius of convergence 1.

   T **F**

8. The Legendre equation admits polynomial solutions for any $n > 0$.

   T **F**

2)Short Answer: (1 point each)

1. Let $f(x) = x + 9$ on $[-\pi, \pi]$. What does the Fourier series of f converge to at 3.

Answer:

2. What is $\int_C \frac{1}{z} dz$ where C is $|z| = 7$ oriented counter clockwise?

Answer:

3. What are the solutions to $z^4 = -1$?

Answer:

4. What type of singularity does $f(z) = e^{\frac{1}{z-3}}$ have at $z = 3$?

Answer:

5. What is the integral of $f(z) = e^z \sin z$ around the curve $|z| = 1$ oriented counter clockwise.

Answer:

6. If $f(x) = 7 \cos 2x$ then what is the sum of the Fourier coefficients of f?

Answer:

7. Determine the radius of convergence of the Taylor series of $f = \frac{1}{(z-1)(z^2+1)}$ centered at -1.

Answer:

8. How many different Laurent series does $f = \frac{1}{(z-1)(z^2+1)}$ have centered at $z = -1$. (Here we consider a Taylor series a Laurent series too.)

Answer:

9. Let $f(x)$ be the function defined, in $[-1,1]$ by

$$f(x) = \begin{cases} x - 1 & -1 \le x < 0 \\ ae^x + 1 & 0 \le x \le 1 \end{cases}$$

Determine a knowing that the Fourier series of $f(x)$ converges to -2 at $x = 0$.

Answer:

3) Compute the following contour integrals. All curves oriented counter clockwise.

1. $\int_C \frac{z^2-4}{z^2+1}\,dz$ where C is $|z-2| = 3.99$.

Answer:

2. $\int_C \frac{e^z}{(z-2)^2}\,dz$ where C is $|z-2| = 7$.

Answer:

3. $\int_C \frac{\tan z}{z}\,dz$ where C is $|z-5| = 5.001$.

Answer:

4) Let $f(z) = \frac{1}{(z-2)(z+3)}$.

1. Determine the regions in which $f(z)$ has a Laurent expansion about $z = 1$.

Answer:

2. Find the Laurent expansion of $f(z)$ about $z = 1$ good in the region containing the point 2.5.

Answer:

5) If $f(x) = 3x - 4$ for $0 \leq x \leq 3$ then find the Fourier sine series of f.

Answer:

6) Compute

1. $\int_{-\pi}^{\pi} \frac{1}{2-\cos x}\, dx$

Answer:

2. $\int_{-\infty}^{\infty} \frac{x^2 \cos(\pi x)}{(x^2+1)(x^2+2)}\, dx$

Answer:

7) Using the Laplace transform solve

$$y'' - 5y' + 4y = 0, \quad y(0) = 0, y'(0) = 2$$

Answer:

8) 1. Show the real part of an analytic function is harmonic.

2.Find the harmonic conjugate $xy - 1$.

Answer:

9) Consider $u_{xx} - 2u_{tt} + 7u_t + u = 0$ subject to the boundary conditions $u(0,t) = 0$ and $u(7,t) = 0$. Find all solutions of the form $u(x,t) = X(x)T(t)$.

Answer:

10) (5 points) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem $y'' + \lambda y = 0$ subject to the boundary conditions $y'(0) = 0, y'(3) = 0$.

Answer:

11) Consider the equation $y'' + 2y' + y = 0$.

1. Find the recurrence relation for the coefficients of a power series solution to this equation.

Answer: