Math 240 Final Exam
Spring 2016

First and Last Name ______________________________(PRINT) Penn ID_______________

Professor **(circle one):** **Panova** **Rimmer** **Powers**

This exam has 6 multiple choice problems and 9 open ended questions. Each question is worth 10 points each for a total of 150 points. Partial credit will be given for the entire exam so be sure to show all work. Circle your answer and give supporting work, a correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam. If you write on the back of any page please indicate this in some way.

You have **120 minutes** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Once you have completed the exam, sign the academic integrity statement below.
Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points	Problem	Points
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	
8			
Total			

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

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Name (printed)

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Signature

1. Let $\mathbf{v}=\begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix}$ and $A=\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix}$. Find the constant k such that $\mathbf{v}^T A\mathbf{v}=0$.

A) -3 B) -2 C) -1 D) 0 E) 1 F) 2 G) 3 H) 4

2. Find the matrix M such that

$$M\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -1 & 0 \\ 6 & -3 & 7 \end{bmatrix}.$$

The sum of the entries in row 2 of M is

A) -3 B) -2 C) -1 D) 0 E) 1 F) 2 G) 3 H) 4

3. The following vectors form a basis for $\mathbb{R}^3$:

$$\mathbf{a}=\begin{bmatrix}1\\2\\3\end{bmatrix},\ \mathbf{b}=\begin{bmatrix}0\\1\\2\end{bmatrix},\ \mathbf{c}=\begin{bmatrix}1\\0\\1\end{bmatrix}.$$

Express the vector $\mathbf{r}=\begin{bmatrix}0\\0\\2\end{bmatrix}$ as a linear combination of $\mathbf{a},\mathbf{b},$ and $\mathbf{c}$ in this manner $\mathbf{r}=s_1\mathbf{a}+s_2\mathbf{b}+s_3\mathbf{c}$.

Find the sum of $s_1+s_2+s_3$.

A) -3 B) -2 C) -1 D) 0 E) 1 F) 2 G) 3 H) 4

4. Suppose R is a 3×3 matrix and $R\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+2y+3z \\ y+2z \\ z \end{bmatrix}$. Then $R^{-1}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} =$

A) $\begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$ B) $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ C) $\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$ D) $\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ E) $\begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$ F) $\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ G) $\begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$ H) None of these

5. Let $A=\begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & k \\ 0 & 1 & -1 \end{bmatrix}$. Find the constant k such that $\det(A)=33$

A) -3   B) -2   C) -1   D) 0   E) 1   F) 2   G) 3   H) 4

6. Find a 2×2 matrix whose eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 6$ and the corresponding eigenvectors are $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

7. Let $T: P_2 \to P_3$ be a linear transformation such that $T(1+x) = x^3 - 2x$, $T(2-x^2) = 2x^3 + x^2 - 2$, and $T(x^2+x) = -x^2 - 2x + 1$. Find $T(1)$ and the matrix $[T]_B^C$, where $B = \{1, x, x^2\}$ and $C = \{1, x, x^2, x^3\}$ are the standard bases for P_2 and P_3 respectively.

8. Let W be the set of 2×2 matrices A such that $A_{2,1}=2A_{1,2}$. W is a subspace of $M_{2\times2}(\mathbb{R})$.

a) Show that $C=\left\{\begin{bmatrix}1&1\\2&1\end{bmatrix},\begin{bmatrix}1&0\\0&1\end{bmatrix},\begin{bmatrix}0&1\\2&3\end{bmatrix}\right\}$ is a basis for W.

b) Find the change of basis matrix $P_{C\leftarrow B}$ where $B=\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix},\begin{bmatrix}0&0\\0&1\end{bmatrix},\begin{bmatrix}0&1\\2&0\end{bmatrix}\right\}$.

9. Diagonalize the following matrix if you know that the characteristic polynomial is $(\lambda-1)^2(\lambda-3)$:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$$

10. Let $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 2a \\ 1 & 0 & 2 & 0 & 1 & a^2 \\ 1 & 2 & 2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$, where a is a parameter.

a) What are the values of a for which $\operatorname{rank}(A) = \operatorname{nullity}(A)$?

b) If $a = -1$, find a basis for the $\operatorname{colspace}(A)$.

11. Find the constant coefficient second order homogeneous differential equation that has solution

$y_c(x) = e^{-x}\left[c_1 \cos(2x) + c_2 \sin(2x)\right]$.

12. Solve the initial value problem

$$y'' + 2y' + y = 2e^{-t} \quad \text{with} \quad y(0) = 1 \text{ and } y'(0) = 0.$$

Find $y(1)$. $y(1) =$

A) $\frac{3}{e}$ B) $\frac{3}{2}$ C) $\frac{4}{e}$ D) $e-1$ E) $\frac{7}{e}$ F) $11-e$ G) $6-\frac{1}{e}$ H) None of these

13. Find the general solution to the differential equation

$$y'' - 2y' + y = \frac{2e^x}{x}.$$

14. Solve the initial value problem for the system of differential equations below.

$$\mathbf{x}' = \begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix} \mathbf{x} \quad \text{with } \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

15. Consider the autonomous system of differential equations

$$\frac{dx}{dt} = x^2 - y$$

$$\frac{dy}{dt} = x - y$$

Find and classify the equilibrium point(s).