MATHEMATICS 240 FINAL EXAM, MAY 5, 2014

C. Pop, S. Shatz

There are 12 problems on this exam, answer all of them. Some are multiple choice, and the first problem consists of ten true/false statements. For the multiple choice questions, **circle the ENTIRE phrase you deem correct among the choices given**. For the true/false statements **circle the ENTIRE word "true" or "false" as the case may be**. There is partial credit given for the answers to the questions numbered 2 through 12–**you must show your work to get any credit for these**. Write answers to ALL questions on the exam paper and show your computations for the problems. There are no blue books just the exam sheets. Credit scores for each problem are indicated with the problem.

No books, tables, notes, calculators, computers (of any sort), cell-phones or any other electronic gear are allowed. One 8 1/2" x 11 1/2" piece of paper, HANDWRITTEN (both sides OK) in your own handwriting, is allowed.

Please fill in the data below NOW.

Your name (print please)——————————————.

Your signature——————————————————.

Circle your instructor's name: Pop Shatz

Your Penn ID number——————————————.

PLEASE DO NOT WRITE BELOW THIS LINE

I	VII
II	VIII
III	IX
IV	X
V	XI
VI	XII

TOTAL

I) True/False (3 points each answer–no work need be shown):

a) If M is a 5 x 5 matrix with eigenvalues 1,1,2,-3,0, then M has an inverse.
True False

b) For a 6 x 6 matrix, M, whose rank is 2, the nullity of M is also 2.
True False.

c) If A is a constant p x p matrix with an eigenvalue λ and a corresponding eigenvector $\vec{v}$, then $\vec{y} = exp(\lambda t)\vec{v}$ is a solution to $\vec{y'} = A\vec{y}$.
True False.

d) The linear equations $A\vec{X} = \vec{b}$, (p equations in q unknowns), have a solution if the rank of the augmented matrix equals rank(A).
True False.

e) Every vector space has a basis.
True False.

f) The rank of A^T equals the rank of A.
True False.

g) If A is a constant 2 x 2 non-singular matrix having eigenvalues $6i$ and $-6i$, then the system of DE's $\vec{y'} = A\vec{y}$ has closed curves as solution trajectories about the equilibrium point $(0, 0)$.
True False.

h) If A is a square matrix, then $AA^T = A^TA$.
True False.

i) The matrix $\begin{pmatrix} 4 & 1 \\ 0 & -4 \end{pmatrix}$ is defective.
True False.

j) When W is a non-empty piece of the vector space V, then W will be a subspace provided $w_1 + w_2$ is in W whenever both w_1 and w_2 are in W.
True False.

II) (15 points–show work) For which value of the number, a, does the system of equations

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ -7 & 5 & 4 & -5 \\ -1 & 8 & 7 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix}$$

have a solution?

1) 11

2) 9

3) 7

4) 5

5) 3.

III) (15 points–show work)   Find e^{tA} where A is the matrix

$$\begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$$

IV) (15 points–show work) What is the general solution to the differential equation: $y''(x) - y'(x) - 2y(x) = 1$?

V) (15 points–show work) A certain mechanical system undergoes motion without an outside forcing term so that the DE of its motion is $y'' + 4y = 0$. If the particle is passing through $y = 1$ when $t = 0$ and $y'(0) = -1$, where is the particle when $t = \frac{3\pi}{2}$?

1) At $y = 0$

2) At $y = -1$

3) At $y = 1$

4) At $y = 2$

5) At $y = -2$.

VI) (15 points–show work) Consider the matrix, A, given by:

$$\begin{pmatrix} 1 & -3 & 2 & 5 & 0 \\ 2 & -2 & 5 & 11 & 2 \\ 0 & 4 & 1 & 1 & 2 \end{pmatrix}.$$

a) Compute rank(A).

b) What is the dimension of the null space of A (that is, the number of basic solutions of the homogeneous equations with A as coefficient matrix)?.

c) What is the dimension of the space spanned by the columns of A?

VII) (15 points–show work) Write P_3 for the vector space of real polynomials of degree ≤ 3.

a) Show that the set, S, consisting of all polynomials of the form

$$p(X) = aX^3 + bX, \text{ with a, b real numbers}$$

is a subspace of P_3.

b) We make a linear transformation, $L : S \longrightarrow \mathbf{R}^2$ defined by

$$L(aX^3 + bX) = (a, a + b).$$

Show that L is invertible.

VIII) (15 points–show work) Find the solution to the initial value problem: $y''(x) - 2y'(x) + 2y(x) = 2x, y(0) = 1, y'(0) = 0.$

IX) (15 points–show work) If we know that $y_1(x) = e^x$ is a solution to the differential equation

$$xy''(x) + (1 - 2x)y'(x) + (x - 1)y(x) = 0,$$

find the general solution of the differential equation.

X) (15 points–show work) Find the general solution of the system of DE's:

$$\vec{y'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{pmatrix} \vec{y}.$$

Write your solution either as a combination of three column vectors or as a 3 x 3 matrix times a constant vector.

XI) (15 points–show work) Consider the space, P_2, of real polynomials of degree ≤ 2 and choose $1, x, x^2$ as a basis for P_2. If T is the linear transformation of P_2 given by: $T(p(x)) = 2p'(x) - p(x)$, write down the matrix of T in the given basis.

XII) (15points–show work) Here are three phase portraits for a system of differential equations: $x' = F(x, y)$; $y' = G(x, y)$.

a) b) c)

Which is correct?

1) The solutions for a) and b) are not bounded for all time, but that for c) is bounded for all time.

2) The solutions for all of a), b), c) are bounded for all time.

3) The solutions for all of a), b), c) are not bounded for all time.

4) The solutions for a) and c) are bounded for all time, but that for b) is not bounded for all time.

5) The solutions for a) and b) are bounded for all time, but that for c) is not bounded for all time.

These pages give further space for computations for the problems.