Penn
UNIVERSITY of PENNSYLVANIA

# Math 240 Final Exam

Dec 19, 2016

No books, paper or any electronic device may be used, other than a hand-written note sheet at most 8.5″ × 11″ in size. Please turn off your cell phones.

This examination consists of twelve (12) long-answer questions, each question is worth 10 points. Please show all your work. Merely displaying some formulas is not sufficient ground for receiving partial credits. Please **box your answers**.

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NAME (PRINTED):

PENN ID:

INSTRUCTOR:

TA:

RECITATION TIME:

My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

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Your signature

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1. Let $A = \begin{bmatrix} 3 & -1 & 1 \\ -7 & 4 & t \\ 2 & 1 & 4 \end{bmatrix}$. For which value of parameter $t$ the column space of $A$ is 2-dimensional? Briefly explain why.

A. 0 B. 1 C. -1 D. -9 E. -2 F. None of these

2. For which values of parameter $t$ the following set of vectors is NOT a basis for $\mathbb{R}^4$? Briefly explain why.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ t \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ t \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 0 \\ t+1 \end{bmatrix} \right\}$$

A. 0 only B. 1 only C. 0 and 1 D. -1 only E. 0 and -1 F. None of these

3. Let $T : P_2 \to P_2$ be a linear transformation defined by $T\,(p(x)) = 2p''(x) + (x-1)p'(x)$, that is $T(a_0 + a_1 x + a_2 x^2) = 4a_2 - a_1 + (a_1 - 2a_2)x + 2a_2 x^2$. Find the matrix of this transformation, $[T]_B^B$, with respect to the standard basis $B = \{1, x, x^2\}$. What is the trace of $[T]_B^B$?

A. 4  B. 0  C. -1  D. 3  E. -2  F. None of these

4. Consider the initial value problem:

$$y'' + 3y' - 10y = 0$$

with $y(0) = 0$ and $y'(0) = 7$. Find $y(1)$.

A. $e^{-2}$ B. $e^{5}$ C. $e^{2} + e^{-5}$ D. $e^{2} - e^{-5}$ E. $1 + e^{7}$ F. None of these

5. Solve the initial-value problem $\vec{x}' = A\vec{x}$ where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

and $\vec{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Find the sum of the components $x_1(t) + x_2(t) + x_3(t)$.

A. $2e^t - \cos(t)$ B. $e^t - 2\sin(t)$ C. $3e^t - 2\cos(t)$ D. $3e^t - 2\cos(t) + \sin(t)$
E. $2e^t - \cos(t) + \sin(t)$ F. None of these

6. Consider the autonomous system

$$\begin{cases} \frac{dx}{dt} = x^2 - y + 2 \\ \\ \frac{dy}{dt} = 3x - y. \end{cases}$$

There are two equilibrium points $(x, y) = (1, 3)$ and $(2, 6)$. Indicate what type of equilibrium points they are. Circle your answers

$(1, 3)$ A. B. C. D. E. $(2, 6)$ A. B. C. D. E.

A. Stable node B. Unstable node C. Saddle D. Stable spiral E. Unstable spiral

7. Let

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$$

Calculate $e^{At}$.

8. Find the general solution to the system $\vec{x}' = A\vec{x}$, where

$$A = \begin{pmatrix} 3 & -1 & -2 \\ 1 & 6 & 1 \\ 1 & 0 & 6 \end{pmatrix}.$$

**Hint**: The only eigenvalue of $A$ is $\lambda = 5$. You can also use the fact that:

$$(A - 5I)^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{pmatrix}.$$

9. Find the general solution to $x^2y'' + xy' + 36y = 0$. , $x > 0$.

10. Consider $4x^2y'' + y = 24\sqrt{x}\ln x$, $x > 0$. $y_1(x) = x^{1/2}$ is a solution for the associated homogenous equation. Find the general solution.

11. Find the general solution to $y'' + 2y' + 5y = 8e^{-x}\cos 2x$.

12. Find the general solution to

$$y'' - 4y' + 4y = \frac{2\ln x}{x} e^{2x}$$

on the interval $x > 0$. (You may use: $\int \ln x dx = x\ln x - x + C$.)

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