Final Exam, Math 240: Calculus III

December 17, 2015

No books, paper or electronic device may be used, other than a hand-written note card at most $4'' \times 6''$ in size. Please turn off your cell phones.

Please show all your work. Make sure to write your final answer in the designated location.

Name (printed): ____________________________________

Instructor: Rimmer Towsner Wibmer

TA: Thomas Brooks Jay Havaldar Konstantinos Karatapanis Philip Pham Tianguang Zhang

Recitation Time: M 8-9 T 8:30-9:30 W 8-9 Th 8:30-9:30 F 8-9 M 9-10 T 9:30-10:30 W 9-10 Th 9:30-10:30 F 9-10

My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

____
Your signature

	1	2	3	4	5	6	First Half Total
Out of	18	10	10	10	10	10	

	7	8	9	10	11		Second Half Total
Out of	10	10	10	10	10		
						Total:	/118

1. Decide whether the following statements are true or false. You do not need to show work.

1. A homogeneous system of 7 linear equations in 5 unknowns always has an infinite number of solutions.

   Circle one: True False

2. Every upper triangular matrix is invertible.

   Circle one: True False

3. If A and B are $n \times n$ matrices with eigenvalues λ and μ respectively then $\lambda - \mu$ is an eigenvalue of $A - B$.

   Circle one: True False

4. The set of all solutions to the vector differential equation

$$\mathbf{x}'(t) = \begin{bmatrix} t & -t & t^2 \\ 1 & -1 & t \\ 1 & 2 & 3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} t \\ 1 \\ 1 \end{bmatrix}$$

   is a vector space of dimension 3.

   Circle one: True False

5. Let $P_1(D)$ and $P_2(D)$ be polynomial differential operators. If y_1 is a solution to $P_1(D)y = 0$ and y_2 is a solution to $P_2(D)y = 0$, then $y_1 + y_2$ is a solution to $P_1(D)P_2(D)y = 0$.

   Circle one: True False

6. If I_n is the $n \times n$ identity matrix and A is an $n \times n$ matrix such that A^2 is the zero matrix, then $I_n + A$ is invertible with inverse $I_n - A$.

   Circle one: True False

7. When using the method of variation of parameters to find the particular solution to a second order differential equation, only one solution to the homogeneous equation is necessary.

   Circle one: True False

8. A polynomial differential operator of order n annihilates every polynomial of degree $n - 1$.

   Circle one: True False

9. If A is a 3×3 nondefective matrix whose eigenvalues are $-2 \pm 3i$ and -1 then every solution to $\mathbf{x}' = A\mathbf{x}$ approaches 0 as t goes to ∞.

   Circle one: True False

2. Compute the inverse of

$$A = \begin{bmatrix} -1 & 1 & 3 \\ 0 & -1 & 2 \\ -1 & 1 & 2 \end{bmatrix}.$$

$A^{-1} =$ ________________

3. Find a basis for the column space of

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 2 & 3 \\ -1 & -4 & 1 & 2 \end{bmatrix}.$$

Make sure to demonstrate that it is actually a basis.

Basis is ____________________

4. If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $\det(A) = -3$, what is

$$\begin{vmatrix} a & g & d \\ b & h & e \\ c & i & f \end{vmatrix}?$$

$$\begin{vmatrix} a & g & d \\ b & h & e \\ c & i & f \end{vmatrix} = \_\_\_\_\_\_\_\_\_\_\_\_$$

5. For each of the following, determine whether or not it is a linear transformation.

(a) $S : P_2 \to \mathbb{R}$ given by $S(p) = p(1)$ (that is, $S(ax^2 + bx + c) = a1^2 + b1 + c$).

(b) $T : M_2(\mathbb{R}) \to \mathbb{R}$ given by $T(A) = \det(A)$.

6. Determine the general solution to $y'' - 5y' + 6y = e^x$.

$y =$ ____________________________

7. Find the general solution of

$$y'' - 6y' + 9y = e^{3x} \ln x$$

using the reduction of order method and the fact that $y_1(x) = e^{3x}$ is a solution to the complementary homogeneous equation. (Hint: remember that $\int \ln x \, dx = x \ln x - x + C$.)

$y =$ ______________________________

8. Solve the initial value problem

$$\mathbf{x}'(t) = \begin{bmatrix} -3 & 1 \\ -1 & -1 \end{bmatrix} \mathbf{x}(t), \ \mathbf{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

$\mathbf{x} =$ ______________________________

9. Determine a basis for the space of real valued solutions of $(D^6 - 2D^5 + 5D^4)y = 0$.

Basis is ______________________________

10. Characterize the equilibrium point of

$$\mathbf{x}'(t) = \begin{bmatrix} -3 & -6 \\ 1 & -7 \end{bmatrix} \mathbf{x}(t)$$

as:

- stable or unstable, and
- node, saddle point, degenerate node, center, or spiral.

Circle one: stable unstable

Circle one: node saddle point degenerate node center spiral

11. Consider the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -1/t & 0 \\ t^2 & 1/t \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 2 \\ -t^3 \end{bmatrix}.$$

Knowing that $\begin{bmatrix} t \\ t \end{bmatrix}$, $\begin{bmatrix} t \\ 3t \end{bmatrix}$, and $\begin{bmatrix} 1/t + t \\ t^2 + t \end{bmatrix}$ are solutions to this system of equations, find the general solution.

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \underline{\hspace{6cm}}$$