UNIVERSITY of PENNSYLVANIA

MATH 240 FINAL EXAM
FALL 2014 MAKE-UP

YOUR NAME:

PROFESSOR'S NAME:

TA'S NAME:

RECITATION NUMBER OR DAY/TIME:

Please *turn off and put away all electronic devices.* You may use both sides of a 8.5″ × 11″ sheet of paper with handwritten notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. **Show all work** on multiple choice questions: one point will be given for clearly circling the correct answer, and up to four points will be giving for the supporting work. Please **clearly mark** a multiple choice option for each problem. Remember to put your name at the top of this page.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

Your signature

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QUESTION NUMBER	POINTS POSSIBLE	YOUR SCORE
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
TOTAL	60	

1. Find the inverse of the matrix A and clearly indicate your answer.

$$A := \begin{bmatrix} 1 & 0 & 2 & 0 \\ 5 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 6 & 0 & 4 & 1 \end{bmatrix}.$$

Circle the option below which equals the nullity (that is, the dimension of the null space) of A.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) none of these

2. For which value of k will the span of the vectors below be two-dimensional?

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ k \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ k \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \\ k \end{bmatrix}, \begin{bmatrix} 0 \\ k-1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2 (f) none of these

3. Consider the 3×3 matrix

$$A := \begin{bmatrix} 0 & 0 & a-1 \\ 3 & a & -5 \\ 4 & 0 & -4 \end{bmatrix}.$$

What value(s) of a make this matrix *non-invertible*?

(a) $a = 0, 1$ (b) $a = 0, 2$ (c) $a = 1, 2$ (d) $a = \pm\sqrt{2}$ (e) $a = 2$ (f) none of these

4. Let $T : V \to V$ be a linear map of a finite dimensional vector space V to itself. The trace of T is defined as the trace of the matrix $\mathfrak{M}_{\mathfrak{B}}^{\mathfrak{B}}(T)$ of T for any basis $\mathfrak{B}$ of V. It is a true fact that this trace is independent of the choice of the basis $\mathfrak{B}$. Let V be the vector space of polynomials of degree at most three, and let T be given by

$$T(P) := x^2 P''' - x^2 P'' - P' + P.$$

Find the trace of T.

(a) -4 (b) -3 (c) -2 (d) -1 (e) 0 (f) none of these

5. In the matrix A from problem 3, let $a = 0$ so that

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 0 & -5 \\ 4 & 0 & -4 \end{bmatrix}.$$

Which of the following ordered lists of vectors is a cycle of generalized eigenvectors corresponding to eigenvalue $\lambda = -2$?

(a) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

(e) $\begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(f) none of these

6. Considering again the matrix

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 0 & -5 \\ 4 & 0 & -4 \end{bmatrix},$$

Find the vector-valued function $X(t)$ satisfying

$$\frac{d}{dt}X(t) = AX(t) \text{ and } X(0) = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}.$$

(a) $X(t) = \begin{bmatrix} 4(t+1)e^{-2t} \\ (14t+4)e^{-2t} \\ 4(2t+1)e^{-2t} \end{bmatrix}$

(b) $X(t) = \begin{bmatrix} 4(t+1) \\ (14t+11) - 7e^{2t} \\ 4(2t+1) \end{bmatrix}$

(c) $X(t) = \begin{bmatrix} 4(2t+1) \\ (14t+11) - 7e^{2t} \\ 4(t+1) \end{bmatrix}$

(d) $X(t) = \begin{bmatrix} 4(t+1)e^{-2t} \\ (14t+11)e^{-2t} - 7 \\ 4(2t+1)e^{-2t} \end{bmatrix}$

(e) $X(t) = \begin{bmatrix} 4(t+1)e^{2t} \\ (14t+11)e^{2t} - 7e^{-t} \\ 4(2t+1)e^{2t} \end{bmatrix}$

(f) none of these

7. Solve the ODE

$$y'' + y' - 6y = 0$$

subject to the initial conditions $y(0) = 0$, $y'(0) = 5$ and clearly indicate your result. Which option below equals $y'''(0)$?

(a) 7 (b) 14 (c) 21 (d) 28 (e) 35 (f) none of these

8. Solve the ODE

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = (4t + 2)e^{-t}$$

subject to the initial conditions $y(0) = -1$, $y'(0) = 1$ and clearly indicate your result. Which option below equals $y(1)$?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2 (f) none of these

9. A weight of 1 g is hanging on the end of a spring suspended inside a fluid-filled container from the top. The spring constant of this spring is $k = 4\,\mathrm{dyn\,cm^{-1}} (= 4\,\mathrm{g\,s^{-2}})$, and the fluid exerts 4 dyn of resistive force for every $1\,\mathrm{cm\,s^{-1}}$ of velocity. The spring is displaced 5 cm from its equilibrium position in the upwards/positive direction and given an initial velocity of $11\,\mathrm{cm\,s^{-1}}$ moving towards the equilibrium position. Find the formula for the displacement from equilibrium as a function of time and clearly indicate the result. At what time (if ever) will the weight first reach the equilibrium position?

(a) 2 s　　(b) 3 s　　(c) 5 s　　(d) 7 s　　(e) 11 s　　(f) none of these

10. Consider the differential equation

$$y'' + (4x+2)y' + (4x^2+4x+2)y = 0.$$

The function $y_1 = e^{-x^2}$ solves this ODE. Use reduction of order to find the general solution. After recording the general soltuion, solve the IVP $y(-1) = 1$, $y'(-1) = 0$ and circle the option below which agrees with your answer.

(a) $y = e^{-x^2}$ (b) $y = e^{-(x-1)^2}$ (c) $y = e^{-(x+1)^2}$

(d) $y = e^{x^2}$ (e) $y = e^{(x-1)^2}$ (f) $y = e^{(x+1)^2}$

11. Let A equal the matrix

$$\begin{bmatrix} 0 & 2 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Compute e^{tA} for all t. Which of the following expressions equals the entry of e^{tA} in row 1 and column 2?

(a) e^t (b) $\cos t$ (c) $\sin t$ (d) $e^t - \cos t + \sin t$ (e) $e^t - \cos t - \sin t$ (f) none of these

12. The following linear autonomous system has two equilibrium points. Describe the types of these points.

$$\frac{dx}{dt} = (y^2 - 1)$$
$$\frac{dy}{dt} = -y + x$$

(a) stable node, center
(b) saddle point, center
(c) saddle point, stable spiral
(d) unstable node, center
(e) sadle point, unstable spiral
(f) degenerate node, center

Scratch Work Page—Will Not Be Graded If Detached