Math 240, Final Exam

December 17, 2013.

Name: ____
TA/Section #: __________

No book, calculator or other devices are allowed. You may have a single 8.5 x 11 sheet in your own handwriting
Show your work and circle the correct answer. No credit need be given for a correct answer with insufficient work

Score		
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
Total	130	

1. The general solution to the ODE:

$$y'' + 5y' + 6y = 0$$

is given by:

A) $c_1 e^{-2x} + c_2 e^{3x}$

B) $c_1 \sin(2x) + c_2 \cos(3x)$

C) $c_1 e^{-2x} + c_2 e^{-3x}$

D) $c_1 e^{-2x} + c_2 x e^{2x}$

E) $c_1 e^{2x} + c_2 e^{3x}$.

2. A trial function for the inhomogeneous ODE:

$$y'' - 4y' + 3y = xe^x + \cos(x)$$

is given by:

A) $Axe^x + Bx^2e^x + C\cos(x) + D\sin(x)$

B) $Ax^2e^x + B\cos(x) + B\sin(x)$

C) $Ae^x + B\cos(x) + Cx\cos(x) + D\sin(x) + Ex\sin(x)$

D) $Ae^x + Bxe^x + Cx^2e^x + D\cos(3x) + E\cos(3x)$

E) $Axe^{-x} + Bx^2e^{-x} + C\cos(x) + D\sin(x)$.

3. Suppose that y solves the following initial value problem:

$$\begin{cases} y'' - 8y' + 7y = 2e^{2x} \\ y(0) = 3, y'(0) = 10. \end{cases}$$

$y(1)$ then equals to:

A) $e^2 + e^3$

B) $e + e^7 + e^2$

C) $e^{-1} + e^7 + e^3$

D) $e + e^4$

E) $e^7 + e^2$.

4. The motion of a spring-mass system is described by:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 11\cos(t) + 7\sin(t).$$

Identify the steady-state part $y_p(t)$ and the transient part $y_c(t)$ of the general solution.

A) $y_p(t) = \sin(t)$ and $y_c(t) = C_1e^t\cos(3t) + C_2e^t\sin(3t)$

B) $y_p(t) = \cos(t)$ and $y_c(t) = C_1e^{-t}\cos(3t) + C_2e^{-t}\sin(3t)$

C) $y_p(t) = \cos(t) + \sin(t)$ and $y_c(t) = C_1e^{-t} + c_2e^{-t}$

D) $y_p(t) = \cos(t) + \sin(t)$ and $y_c(t) = C_1e^{-t}\cos(3t) + C_2e^{-t}\sin(3t)$.

E) $y_p(t) = \sin(t) + t\sin(t)$ and $y_c(t) = C_1e^{-t}\cos(3t) + C_2e^{-t}\sin(3t)$.

5. Consider the ODE:

$$x^2y'' - 5xy' + 9y = 0,\ x > 0.$$

Find a solution to the ODE of the form $y_1(x) = x^r$ for some real number r and then use the method o reduction of order to find a second linearly independent solution $y_2(x)$. The solutions y_1 and y_2 can be chosen as:

A) $y_1(x) = \frac{1}{x}$ and $y_2(x) = \ln(x)$

B) $y_1(x) = x^3$ and $y_2(x) = x^2 \cdot \ln(x)$

C) $y_1(x) = x^2$ and $y_2(x) = x^3$

D) $y_1(x) = x^2$ and $y_2(x) = x^2 \cdot \ln(x)$

E) $y_1(x) = x^3$ and $y_2(x) = x^3 \cdot \ln(x)$.

6. Describe the type of the first quadrant (non-zero) equilibrium point of the linear autonomous system

$$\frac{dx}{dt} = y(x-4)$$
$$\frac{dy}{dt} = x - 4y^2$$

A) stable node B) unstable node C) saddle point D) center E) spiral F) degenerate node

7. Let $\mathbf{x}(t)$ be a 4×1 vector function satisfying

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 2 & 1 & 0 \\ -2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix} \mathbf{x}(t)$$

and

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} .$$

Find $\mathbf{x}(\pi)$.

A) $\begin{bmatrix} 1 \\ 0 \\ -\pi \\ 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 \\ 0 \\ \pi \\ 0 \end{bmatrix}$ C) $\begin{bmatrix} 0 \\ 1 \\ 0 \\ -\pi \end{bmatrix}$ D) $\begin{bmatrix} 0 \\ 1 \\ 0 \\ \pi \end{bmatrix}$ E) $\begin{bmatrix} -1 \\ 0 \\ -\pi \\ 0 \end{bmatrix}$

8. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

Find the upper right entry of the exponential matrix function e^{At}.

A) $\frac{e^t+e^{3t}}{2}$ B) $\frac{-e^t+e^{3t}}{2}$ C) $\frac{-e^t-e^{3t}}{2}$ D) $\frac{e^t-e^{3t}}{2}$ E) $\frac{e^t-e^{-3t}}{2}$

9. Find the dimension of the eigenspace of the eigenvalue 2 of the matrix

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

A) 0 B) 1 C) 2 D) 3 E) 4