Math 240 **FINAL EXAM** **May 9, 2008**

Circle one: **Professor Popa**
Professor Storm
Professor Ziller

Name: ______________________________

Penn Id#: ____________________

Signature: ____________________________

TA: ____________________________

Recitation Day and Time: ____________________

You need to show your work, even for multiple choice problems. A correct answer with no work will get 0 points. The only exception are True/False problems, where no work needs to be shown. Each problem is worth 10 points.

You are NOT allowed to use a calculator. The extra double sided sheet of paper needs to be hand written in your own hand writing (no copies allowed).

(Do not fill these in; they are for grading purposes only.)

1)	9)
2)	10)
3)	11)
4)	12)
5)	13)
6)	14)
7)	15)
8)	

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Total

1. The matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

has $K = \begin{pmatrix} 1 \\ 6 \\ -13 \end{pmatrix}$ as an eigenvector. What is the corresponding eigenvalue?

2. The matrix

$$A = \begin{pmatrix} 8 & -2 & 2 \\ -2 & 5 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

has eigenvalues $\lambda_1 = 0$ and $\lambda_2 = 9$ (with multiplicity 2). An eigenvector corresponding to λ_1 is $v_1 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and an eigenvector corresponding to λ_2 is $v_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$. Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal.

3. Compute the determinant

$$\begin{vmatrix} 1 & 2 & 1 & 4 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{vmatrix}$$

Answer:

(a) 2 (b) 4 (c) -4 (d) 8 (e) -2 (f) -8

4. For each of the following statements, determine whether they are true or false. All the matrices below are assumed to have real entries. No work needs to be shown for this problem.

(a) Any orthogonal 3x3 matrix has an eigenvalue equal to 1 or -1 . True○ False○

(b) There is a symmetric matrix having i and $-i$ as eigenvalues. True○ False○

(c) If A is any 2x2 matrix of rank 1, then the system True○ False○
$AX = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ has infinitely many solutions.

(d) If A is any 3x2 matrix of rank 2 and B a 2x3 matrix True○ False○
such that $AB = 0$, then $B = 0$.

(e) If A is a 2x2 matrix with $A^2 = I$ (the identity matrix) True○ False○
then $A = \pm I$.

5. Consider the vector field

$$\mathbf{F} = \left(\frac{-z^2}{5} - z + \pi y e^{\sin x} \cos x \right) \mathbf{i} + \left(\pi e^{\sin x} - x \right) \mathbf{j} - \frac{2xz}{5} \mathbf{k}$$

and the curve C given by

$$(2\cos t, 2\sin t, 0)$$

for $-\pi/2 \le t \le \pi/2$. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

(a) $2\pi\sqrt{2}$ (b) 0 (c) 4π (d) $-\pi$ (e) -2π (f) 2π (g) none of the above

6. In the following true/false problems, $\mathbf{F}$ is any vector field in 3-dimensions and f is any function in 3 variables. (You may assume $\mathbf{F}$ and f have continuous derivatives.) You do not need to show any work. For each problem, state whether the given identity is true or false.

(a) $\operatorname{div}(\nabla f) = 0$	True◯	False◯
(b) $\operatorname{curl}(\nabla f) = \mathbf{0}$	True◯	False◯
(c) $\operatorname{div}(\operatorname{curl}\mathbf{F}) = 0$	True◯	False◯
(d) $\operatorname{curl}(\operatorname{curl}\mathbf{F}) = \mathbf{0}$	True◯	False◯
(e) $\nabla(\operatorname{div}\mathbf{F}) = \mathbf{0}$	True◯	False◯

7. Define the function

$$f(x,y,z) = e^{(\sin x \cdot \cos y)} \cdot \left(z + \frac{\pi}{2}\right).$$

Let C be the curve

$$\big(t\,\cos^2(2t),\ t\,\sin(t),\ t\big)$$

for $0 \le t \le \pi/2$. Compute the integral

$$\int_C \frac{\partial f}{\partial x}\,dx + \int_C \frac{\partial f}{\partial y}\,dy + \int_C \frac{\partial f}{\partial z}\,dz.$$

8. Let S be the closed surface in 3-space formed by the cone

$$x^2 + y^2 - z^2 = 0, \quad 1 \leq z \leq 2,$$

the disk $x^2 + y^2 \leq 4$ in the plane $z = 2$, and the disk $x^2 + y^2 \leq 1$ in the plane $z = 1$. Define the vector field

$$\mathbf{F}(x, y, z) = xy^2\mathbf{i} + x^2y\mathbf{j} + \sin x\,\mathbf{k}$$

and let $\mathbf{n}$ be the outward pointing unit normal vector to S. Compute the surface integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS.$$

9. Find the solution of the differential equation $y' - y = y^2$ with $y(0) = \frac{1}{3}$.

10. Let y be the solution of $y'' = e^{-3t} - y'$ that passes through the origin and has a horizontal tangent line there. Then $\lim_{t\to\infty} y(t)$ is equal to:

Answer:

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) -2 (e) $-\frac{1}{4}$

11. Let $y(t) = (y_1(t), y_2(t))$ be any non-zero solution of the system of differential equations

$$\begin{aligned} y_1' &= y_1 + 2y_2 \\ y_2' &= 3y_1 + 2y_2 \end{aligned}$$

such that $\lim\limits_{t\to\infty} y(t) = 0$. Then $\frac{y_1(1)}{y_2(1)}$ is equal to

Answer:

(a) -1 (b) 1 (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$ (e) $\frac{3}{2}$

12. Find the solution of $xy'' + y' = -\frac{y}{x}$ with $y(1) = 0$, $y'(1) = 2$. Do not use a power series approach.

13. A spring satisfies the differential equation $x'' + 16x = 0$. It is released one meter above its equilibrium position with a downward velocity of 3 meters per second. What is its highest position above the equilibrium position?

14. Find an inhomogenous linear second order differential equation with constant coefficients having $y_p = \frac{1}{2}x^2 - x$ as a particular solution, and $y_1 = 3$ and $y_2 = e^{2x}$ as solutions of the associated homogeneous differential equation.

15. Consider the differential equation $2xy'' + y' + y = 0$.

a) Show that the equation has two *linearly independent* series solutions. You do not have to compute the coefficients of the series.

b) Find the coefficient a_2 in a *power series solution* $y(x) = \sum_{n=0}^{\infty} a_n x^n$ with $y(0) = 1$, $y'(0) = -1$.