Math 240, Final Exam

May 6, 2004

Name: ______________________________

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This exam consists of 15 pages. **In order to receive full credit you need to show all your work** .

Score		
1	5	
2	5	
3	5	
4	5	
5	5	
6	10	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
13	5	
14	5	
Total	75	

1. Show that the differential equation $(2x+y^3)dx+(3xy^2-e^{-2y})dy=0$ is exact and find the particular solution with $y(-1)=0$.

2. Find the general solution to the differential equation

$$y'' - 2y' + 2y = \sin(x)$$

3. For what values of λ is every non-zero solution of the differential equation

$$y^{''} + \lambda y' + y = 0$$

unbounded ?

4. For what values of λ does the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & \lambda & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

have rank 3 ?

5. Determine whether the statement is true or false. If true, give a justification, if false, give a counterexample.

Let A be an $n \times n$ matrix with distinct positive eigenvalues. Then

1. det(A) > 0
2. A is diagonalizable
3. A is orthogonal
4. A is symmetric
5. Eigenvectors of A corresponding to distinct eigenvalues are orthogonal.

6. Find a basis of eigenvectors for the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

7. Let T be the following matrix

$$T = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

Use row operations to show that $\det T = (b - a)(c - a)(c - b)$.

8. While subject to force $\mathbf{F}(x, y) = y^3\mathbf{i} + (x^3 + 3xy^2)\mathbf{j}$, a particle travels once around the circle of radius 3. Use Green's Theorem to find the work done by $\mathbf{F}$.

9. Evaluate

$$\int_C (1 - ye^{-x})dx + e^{-x}dy$$

where C is the curve $r(t) = (e^{cos\pi t}, 1/(t^7 + 1)),\ 0 \leq t \leq 1.$

10. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot \mathbf{dr}$, where

$$\mathbf{F}(x, y, z) = 2y\mathbf{i} + 3z\mathbf{j} + x\mathbf{k}$$

and C is the triangle with vertices $(0, 0, 0)$, $(0, 2, 0)$ and $(1, 1, 1)$ oriented so that the vertices are traversed in that order.

11. Let Q be the solid bounded by the cylinder $x^2 + y^2 = 4$, the plane $x + z = 6$ and the xy-plane. Find $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where S is the surface of Q oriented by the outside normal, and $\mathbf{F}(x, y, z) = (x^2 + \sin z)\mathbf{i} + (xy + \cos z)\mathbf{j} + e^y \mathbf{k}$.

12. Determine whether the set of solutions of the equation $3x + 2y = 5$ forms a subspace of $\mathbb{R}^2$ under vector addition and scalar multiplication.

13. a) Give an example of a vector field $\mathbf{F}$ on $\mathbb{R}^2 - (0,0)$, where it has continuous partials, such that $\nabla \times \mathbf{F} = 0$ and $\int_C \mathbf{F} \cdot dr$ is not independent of the path C joining a given set of points.

b) Now suppose that $\mathbf{F}$ is defined on $\mathbb{R}^3 - (0,0,0)$ where it has continuous partials, and $\nabla \times \mathbf{F} = 0$. What can you say about the path-independence of the corresponding line integral ?

14. Find the simple closed curve C oriented counterclockwise on which the line integral

$$\int_C (y^3 - y)dy - 2x^3 dx$$

achieves its maximum value.