SOME FORMULAS

Volume of a solid sphere of radius r: $\frac{4}{3}\pi r^3$
Surface area of a sphere of radius r: $4\pi r^2$
Volume of a cylinder: $\pi r^2 h$
Volume of a cone: $\frac{1}{3}\pi r^2 h$

FROM TRIGONOMETRY

$sin^2\alpha = \frac{1}{2}(1 - cos2\alpha)$
$cos^2\alpha = \frac{1}{2}(1 + cos2\alpha)$

This exam contains 20 pages

Math 240, Final Exam May 1, 2003

Name: ______________________________

Instructor: ______________________________

Score		
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
Total	180	

1. Indicate whether or not the following expressions are defined:

1. $A + B$, where A is a 5×5 matrix, B a 5×6 matrix.
2. The dot product of $\mathbf{v}$ and $\mathbf{w}$, where $\mathbf{v} = [0, 1, 2]$ and $\mathbf{w} = [2, 5, 7]$.
3. The matrix product of $\mathbf{v}$ and $\mathbf{w}$, where $\mathbf{v} = [0, 1, 2]$ and $\mathbf{w} = [2, 5, 7]$.
4. $det(C)$, where C is a 2×3 matrix.
5. $rank(C)$, where C is a 2×3 matrix.

2. True or false:

1. The set of vectors $[a, b, c, d]$ with $a \geq 0$ form a vector space.
2. The set of vectors $[a, b, c, d]$ with $c = 0$ form a vector space.
3. $Ax = v$ always has a solution $x \neq \mathbf{0}$ if A is a 7×5 matrix, x is 5×1 vector and v a non -zero 7×1 vector.

3. First find the general solution of

$$y'' - y' - 6y = 0$$

Next compute the solution y of

$$y'' - y' - 6y = 12x, \quad y(0) = \frac{1}{3}, \quad y'(0) = 0$$

Then $y(1)$ is equal to

a) $\frac{2}{5}(e^3 - e^2) - \frac{5}{3}$ *b*) $\frac{2}{5}(e^3 - e^{-2}) - \frac{1}{3}$ *c*) $\frac{2}{5}(e^{-3} - e^2) - \frac{5}{3}$

d) $\frac{2}{5}(e^{-3} - e^{-2}) - \frac{1}{3}$ *e*) $\frac{2}{5}(e^3 - e^{-2}) - \frac{5}{3}$ *f*) $\frac{2}{5}(e^3 - e^2) - \frac{1}{3}$.

4. The value of t such that the matrix

$$\begin{bmatrix} 1 & t & 2 \\ 0 & 4 & t \\ 3 & -5 & 4 \end{bmatrix}$$

has no inverse is

$a)$ -1 $b)$ 0 $c)$ 1 $d)$ 2 $e)$ 3

5. Suppose that $F = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$, M and N have continuous partial derivatives and C is a smooth closed curve enclosing a region D.
Indicate whether each expression is defined and for those which are defined, label each statement as true or false:

a) If $\iint_D (N_x - M_y) = 0$, then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$

b) div $(\mathbf{F})$ is a vector.

c) If $\mathbf{F} = \nabla f$ and E is a curve starting at P_1 and ending at P_2, then: $\int_E \mathbf{F} \cdot \mathbf{r} = \mathbf{F}(P_1) - \mathbf{F}(P_2)$

d) $curl(div(\mathbf{F})) = 0$.

6. The direction of the steepest ascent at $P = (3, 0)$ of the mountain $f(x, y) = 4 - \frac{2}{3}\sqrt{x^2 + y^2}$ is:

a) $\mathbf{i} + \mathbf{j}$ *b*) $-\dfrac{2}{3}\mathbf{i}$ *c*) $-\dfrac{2}{3}\,\mathbf{i} + \mathbf{j}$ *d*) 0 *e*) $-\dfrac{2}{3}\mathbf{j}$ *f*) $-\dfrac{2}{3}\,\mathbf{j}$

7. One eigenvalue of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is 1. Which of the following equations gives the corresponding eigenspace (i.e., the space that contains all eigenvectors for the eigenvalue 1)?

$a)\quad x_2 = 0 \qquad b)\quad x_1 + x_2 + x_3 = 0 \qquad c)\quad -x_1 + x_2 = 0$

$d)\quad x_1 + 2x_2 + x_3 = 0 \qquad e)\quad x_1 + x_3 = 0 \qquad f)\quad x_1 + 2x_2 = 0$

8. The value of the line integral $\int_C xy^2dx + x^2ydy$ over the curve C parametrized by $(1 + cos^3(t))\mathbf{i} + (1 - sin^6(t))\mathbf{j}, \quad 0 \leq t \leq \pi$, and oriented in the direction of increasing t is:

$a)\ \ \pi/2 \qquad b)\ \ -\pi \qquad c)\ \ 2 \qquad d)\ \ -2 \qquad e)\ \ 0 \qquad f)\ \ -1 \qquad g)\ \ -\pi^2/4$

9. Let $y(x)$ be the solution of the following initial value problem

$$y' + x^2 y = 3x^2, \quad y(0) = 1$$

Then $y(1)$ is equal to

$a)$ 0 $\quad b)$ 1 $\quad c)$ $e^{-1/3}$ $\quad d)$ $3 + 2e^{-1/3}$

$e)$ $3 - 2e^{-1/3}$ $\quad f)$ $e - 1$ $\quad g)$ $e + 1$.

10. The following matrix is orthogonal

$$\begin{bmatrix} a & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ b & \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} \\ c & \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} \end{bmatrix}$$

if $[a, b, c]$ is

$a)\quad \dfrac{1}{\sqrt{3}}[0,0,1],\qquad b)\quad \dfrac{1}{\sqrt{2}}[1,1,0],\qquad c)\quad [0,0,0]\qquad d)\quad [1,0,0]$

$e)\quad [1,1,0]\qquad f)\quad [2,2,1]\qquad g)\quad \dfrac{1}{\sqrt{3}}[2,2,1].$

11. Suppose $M(x, y)$ is a smooth function on the xy-plane and N is a constant. Under what conditions is $Mdx + Ndy$ exact? Label each statement as true or false:

a) For any function M and any constant N.

b) If $\frac{\partial M}{\partial y} = 0$.

c) If $\frac{\partial M}{\partial x} = 0$.

d) If there is some function $u(x, y)$ such that $\frac{\partial u}{\partial x} = M$ and $\frac{\partial u}{\partial y} = N$.

e) If there is some function $u(x, y)$ such that $\frac{\partial u}{\partial x} = N$ and $\frac{\partial u}{\partial y} = M$.

12. Let $x^T = [x_1, x_2, x_3, x_4]$ be a solution of $Ax = [0, 0, 0, 0]^T$, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Then $x_1 + x_2 + x_3 + x_4$ is equal to

$a)$ 4 $b)$ 3 $c)$ 2 $d)$ 1 $e)$ 0 $f)$ -1 $g)$ -2

13. Find the solution $y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$ of the system of differential equations

$$y_1' = 6y_1 + 9y_2$$

$$y_2' = y_1 + 6y_2$$

$$y_1(0) = 3, \quad y_2(0) = 3$$

Then $y_1(1) + y_2(1)$ is equal to

$a)$ $3e^9 + 2e^3$ $\quad b)$ $8e^9 - 2e^3$ $\quad c)$ $-4e^9 + 2e^3$

$d)$ $8e^9 + 2e^3$ $\quad e)$ $4e^9 - 2e^3$ $\quad f)$ $4e^9 + 2e^3$

14. The surface integral $\iint_S G(r)\, dA$, where $G(r) = xy + x^2$ and S is the surface given by $x^2 + y^2 = 1$, $|z| \le 2$ is equal to

$a)$ 0 $b)$ -8π $c)$ -4π $d)$ -2π
$e)$ 2π $f)$ 4π $g)$ 8π.

15. Let P_1 and P_2 be two points in three-space, and C a curve joining P_1 to P_2. For what values of a is the line integral $\int_C 3x^2y^5dx+ax^3y^4dy+dz$ independent of C ?

$a)$ 5 $\quad b)$ -5 $\quad c)$ 3 $\quad d)$ -3 $\quad e)$ 1 $\quad f)$ 0 $\quad g)$ *novalues*

16. The value of the line integral $\int_C (-y + cos(x^2))dx + (3x + e^{\sqrt{y^2-1}})dy$ where C is the boundary of the rectangle with vertices at $(1,0), (1,3), (5,0), (5,3)$ oriented counterclockwise is:

$a)$ 12 $b)$ 15 $c)$ 24 $d)$ 12π $e)$ 48 $f)$ -6π $g)$ 0

17. The value of the surface integral $\iint_S F \cdot n \, dA$, where $F = \frac{x^3}{3} i + \frac{y^3}{3} j + z^2 k$, and S is the closed cylindrical shell $x^2 + y^2 = 4, 0 \leq z \leq 3$ (including the top and bottom disks), oriented by the outwards normal is:

$a)$ 0 $b)$ 60π $c)$ -60π $d)$ 12π $e)$ -12π $f)$ 10 $g)$ 36π

18. Let $F = \mathbf{i} + 3x\mathbf{j} + e^{sin(2x)}\mathbf{k}$ be a vector field. The value of $\int\int_S (\nabla \times F) \cdot n dA$ over the surface $z = x^2 + y^2 - 9, z \leq 0$, oriented by the normal pointing "upward" (i.e. in the positive z direction) is:

$a)$ 3π $\quad$ $b)$ 27π $\quad$ $c)$ 6π $\quad$ $d)$ 0π $\quad$ $e)$ -9π $\quad$ $f)$ 9π $\quad$ $g)$ $e^{2\pi}$