Problem:	1	2	3	4	5	6	7	8	9	10	11	12
Score:												

Total:

Please do not write above this line

UNIVERSITY OF PENNSYLVANIA
DEPARTMENT OF MATHEMATICS

MATH 240 FINAL EXAM

Wednesday December 14, 2011
9 AM - 11 AM

Your name (printed) ____

Signature____

Professor (circle one): Chris Jankowski David Lipsky Herman Gluck

TA___________________Recitation Day________Recitation hour_______

INSTRUCTIONS. There are 12 problems on this exam, each worth 25 points, with partial credit available.

To get full credit for a problem, you must get the right answer and your written supporting work must be understandable by us (the graders) and be sufficient in our opinion to fully derive your answer. Even if you can do parts of the problem in your head, you must show all the work.

In particular, a problem with a correct answer may receive full, partial or no credit.

You must put all answers in the spaces provided.

There are two blank sheets at the end of the exam for extra work space, as needed.

If you are in doubt about the meaning of a problem, just raise your hand during the exam and ask...we will try to help if we can.

No books, notes, calculators, computers or phones during the exam...except that you may use one 8 1/2 by 11 inch sheet of paper with information written or typed on both sides.

This exam is conducted under Penn's Code of Academic Integrity.

Please write **dark** and **large**, and print your name on each sheet, in the space provided at the top.

Good luck!

Problem 1. Solve the given system or show that no solution exists:

$$
\begin{aligned}
x + 2y \quad &= 1 \\
3x + 2y + 4z &= 7 \\
-2x + y - 2z &= -1
\end{aligned}
$$

Answer. x = ______ , y = ______ , z = ______ .

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Problem 2. Suppose that A is a 3×3 matrix with eigenvalues $\lambda_1 = -1$, $\lambda_2 = 0$ and $\lambda_3 = 1$, and corresponding eigenvectors

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(a) Find the matrix A . (b) Find the matrix A^{20} .

Answer. A = ______________________ A^{20} = ______________________

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Problem 3.

For each of the following, answer TRUE or FALSE.
If the statement is false in even a single instance, then the answer is FALSE.
There is no need to justify your answers to this problem.

(a) If A is an invertible 4×4 matrix, then $(A^T)^{-1} = (A^{-1})^T$, where A^T denotes the transpose of A.

(b) If A and B are 3×3 matrices with $\text{rank}(A) = \text{rank}(B) = 2$, then $\text{rank}(AB) = 2$.

(c) If A and B are invertible 3×3 matrices, then $A + B$ is invertible.

(d) If A is an $n \times n$ matrix with rank less than n, then for any vector $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions.

(e) If A is an invertible 3×3 matrix and λ is an eigenvalue of A, then $1/\lambda$ is an eigenvalue of A^{-1}.

Answers (Please circle one in each case).

(a) True / False (b) True / False (c) True / False (d) True / False (e) True / False

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Problem 4. Diagonalize the matrix A =

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix},$$

by finding the eigenvalues of A listed in increasing order, corresponding eigenvectors, a diagonal matrix D and a matrix P such that $A = P D P^{-1}$.

You must put the following answers in the designated spaces:

(1) Eigenvalues of A in increasing order: _____ , _____ and _____

(2) Eigenvectors of A in order corresponding to eigenvalues above:

_____________ , _____________ and _____________ .

(3) Diagonal matrix D =

(4) Matrix P =

Extra page for Problem 4.

Problem 5. Find the value of the line integral

$$I = \int_C (3\pi x^2 y + ye^x)\, dx + (\pi x + \pi x^3 + e^x)\, dy$$

where C is the curve parametrized by $x = \sin t$, $y = t$
for $0 \le t \le \pi$, and oriented in the direction of increasing t.

Answer. $I =$ __________

Problem 6. Let S be the square with vertices

$$(1, 0, 0)\ ,\ (0, 1, 0),\ (0, 1, \sqrt{2})\ ,\ (1, 0, \sqrt{2})\ ,$$

and let C be the boundary of S, traversed in this order of vertices.

Let W be the vector field $W = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$.

Find the value of the integral $I = \int_C W \bullet d\mathbf{r} = \int_C z\,dx + x\,dy + y\,dz$.

Answer. I = ___________

Problem 7. Evaluate

$$I = \int_C \; y\,(x^2 + y^2)^{-1}\,dx \; - \; x\,(x^2 + y^2)^{-1}\,dy$$

where C is the boundary of the square with vertices at
(2, −2), (2, 2) , (−2, 2) and (−2, −2) , traversed counterclockwise.

Answer. I = __________

Problem 8. Consider the vector field

$$W = x^3y^2\,\mathbf{i} - x^2y^3\,\mathbf{j} + (1 + z)\,\mathbf{k}\,.$$

Find the outward flux of W through the portion S of the paraboloid $z = 4 - x^2 - y^2$ which lies above the xy-plane.

Answer. Outward flux of W through S = __________

Problem 9. Solve the differential equation

$$x^2 y'' - x y' - 15 y = 0$$

for the unknown function $y(x)$, subject to the initial conditions
$y(1) = 3$ and $y'(1) = 7$.

Answer. $y(x)$ = ____________________

Problem 10. Find the general solution $y(x)$ to the nonhomogeneous equation

$$y'' + y' - 6y = 10\, e^{2x}\,.$$

Answer. $y(x) =$ ______________________

Problem 11. Solve the system of differential equations

$$dx/dt = 2x + y$$

$$dy/dt = x + 2y$$

for unknown functions $x(t)$ and $y(t)$, subject to the initial conditions $x(0) = 1$ and $y(0) = 5$.

Answer. $x(t) =$ __________________ and $y(t) =$ __________________

Problem 12. Suppose a solution of the nonhomogeneous differential equation

$$y'' + y' - 2xy = e^x$$

has the form $y(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots$.

If $c_0 = 1$ and $c_1 = -1$, what is c_4 ?

Reminder. The power series for e^x is

$$e^x = 1 + x + x^2/2! + x^3/3! + \dots + x^n/n! + \dots$$

Answer. c_4 = __________

Extra space for your work.

Please put the problem number here, and refer forward to this page 15 on the original problem page.

Extra space for your work.

Please put the problem number here, and refer forward to this page 16 on the original problem page.

END OF EXAM