Math 240, FINAL EXAM

December 18, 2012

INSTRUCTIONS	OFFICIAL USE ONLY

INSTRUCTIONS:

1. Please complete the information requested below. There are 9 multiple choice problems and 1 True/False problem. Partial credit will be given on the multiple choice questions.

2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.

4. You are allowed to use one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

• **Name** (please print):

• **Name of your Professor:**
◯ **Ryan Blair** ◯ **Vasile Brinzanescu**
◯ **Tony Pantev**

• **Name of your TA:**
◯ **Shiying Dong** ◯ **Ryan Eberhart**
◯ **Ryan Manion** ◯ **Sebastian Moore**

• **Recitation day and time:**

• **My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this exam.**
Signature:

Problem	Points
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
Total	

(1) $\boxed{\text{10 POINTS}}$ Find the outward flux of the vector field $\overrightarrow{F} = x\widehat{\boldsymbol{\imath}} + y\widehat{\boldsymbol{\jmath}} + z\widehat{\boldsymbol{k}}$ through the boundary of the solid region R bounded by the paraboloid $z = 4 - x^2 - y^2$ and the plane $z = 0$.

(a) 1

(b) 6π

(c) 24π

(d) 2

(e) $\pi\sqrt{2}$

(f) $\dfrac{2\pi}{3}\left(\sqrt{2} - 1\right)$

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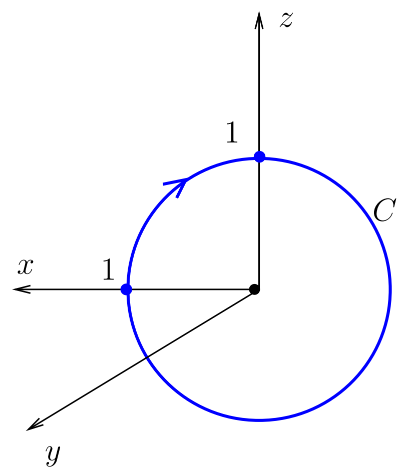
(2) $\boxed{\text{10 POINTS}}$ Let C be the parametrized curve $\overrightarrow{r}(t) = \langle \cos(t), 0, \sin(t) \rangle$ for $0 \leq t \leq 2\pi$. Find the line integral $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$ for the vector field

$$\overrightarrow{F} = \langle -2z + \sin(e^x),\ 3y + 2,\ 2x + 3y \rangle.$$



(a) $\sin(e^{\pi})$

(b) 8π

(c) $3\pi + 2$

(d) 4π

(e) $\sin(e^{2\pi}) + 1$

(f) 0

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(3) $\boxed{\text{10 POINTS}}$ Consider the homogeneous linear system $Ax = 0$, where

$$A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 1 & 3 & -2 & -2 \\ 0 & 0 & 2 & 3 \end{bmatrix}.$$

Which of the following statements is **correct**.

(a) $Ax = 0$ has no solutions.

(b) $Ax = 0$ has one free variable.

(c) $Ax = 0$ has two free variables.

(d) $Ax = 0$ has three free variables.

(e) $Ax = 0$ has a unique solution.

(f) x_1 and x_2 are pivot variables for $Ax = 0$.

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(4) $\boxed{\text{10 POINTS}}$ Which **one** of the following is **not** a basis for the vector space of all symmetric 2×2 matrices. Justify that the set you pick is not a basis.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -2 & -2 \\ -2 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

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(5) $\boxed{\text{10 POINTS}}$ Which of the following collections of matrices form a subspace in the vector space $\mathrm{Mat}_{3\times 3}(\mathbb{R})$.

(i) All matrices of rank 1.

(ii) All matrices A satisfying $2A - A^T = 0$.

(iii) All matrices A satisfying

$$A \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(a) (i) and (ii) only

(b) (i) and (iii) only

(c) (ii) and (iii) only

(d) (i) only

(e) (ii) only

(f) (iii) only

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(6) $\boxed{\text{10 POINTS}}$ Which of the following matrices have two linearly independent eigenvectors. Circle **all** that apply and justify your answers.

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$

(e) $\begin{pmatrix} 3 & 0 \\ 1 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 3 & 0 \\ 1 & -3 \end{pmatrix}$

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(7) $\boxed{\text{10 POINTS}}$ Please circle "**T**" for true or "**F**" for false in the space provided to the left of the following statements. You **do not** need to justify your answer for full credit.

T **F** The sum of two solutions to a linear, non-homogeneous differential equation is always a solution.

T **F** Three column vectors in $V_2(I) = \mathrm{Fun}(I, \mathbb{R}^2)$ must be linearly dependent.

T **F** The fundamental solution set to a vector differential equation given by $\mathbf{x}' = A\mathbf{x}$ forms a spanning set for the vector space of solutions to $\mathbf{x}' = A\mathbf{x}$.

T **F** The zero function is a solution to every linear differential equation.

T **F** An initial value problem for an n-th order, homogeneous, constant coefficient, linear differential equation always has n linearly independent solutions.

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(8) $\boxed{\text{10 POINTS}}$ Let $y(x)$ be the solution to the initial value problem

$$y'' + 2y' + 2y = 0, \quad y(0) = 1,\ y'(0) = 1.$$

What is $y(\pi)$?

(a) $-e^{\pi}$

(b) 0

(c) $2e^{\pi}$

(d) $-e^{-\pi}$

(e) $1 + e^{\pi}$

(f) $1 - 2e^{-\pi}$

(9) $\boxed{\text{10 POINTS}}$ Let $y(x)$ be the general solution of the linear differential equation

$$y'' - 6y' + 9y = e^x + 1.$$

Find $\lim_{x \to -\infty} y(x)$.

(a) 4

(b) $\frac{1}{4}$

(c) $\frac{1}{9}$

(d) -6

(e) -9

(f) $e^3 + 1$

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(10) $\boxed{\text{10 POINTS}}$ Let $\mathbf{x}$ be the solution of the system of differential equations

$$\mathbf{x}' = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \mathbf{x}$$

which satisfies the initial condition $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. What is $x_3(1)$?

(a) $-e$

(b) $2e$

(c) $-3e^2$

(d) e^2

(e) $e - e^2$

(f) $2e^2 - e$