Problem:	1	2	3	4	5	6	7	8	9	10	11	12
Score:												

Total:

Please do not write above this line

UNIVERSITY OF PENNSYLVANIA
DEPARTMENT OF MATHEMATICS

MATH 240 FINAL EXAM

Monday December 21, 2009

Your name (printed) ____

Signature____

Professor (circle one): Patrick Clarke Chenxu He Herman Gluck

TA_______________Recitation Day________Recitation hour_______

INSTRUCTIONS. There are 12 problems on this exam; problem 4 is worth 12 points, and all the rest are worth 8 points each.

For the 8 point problems, to get full credit you must get the right answer and your written supporting work must be understandable by us (the graders) and be sufficient in our opinion to fully derive your answer. Even if you can do the problem in your head, you must show all the work. For these problems, half credit is available if your work shows, in our opinion, substantial progress towards the solution.

For the 12 point problem, partial credit is available.

Note. A problem with a correct answer may receive full, partial or no credit.

You must put all answers in the spaces provided.

No books, notes, calculators or computers during the exam...except that you may use one 8 1/2 by 11 inch sheet of paper with information written or typed on both sides.

This exam is conducted under Penn's Code of Academic Integrity.

Please write **dark** and **large**, and print your name on each sheet, in the space provided at the top.

Good luck!

Problem 1. Solve the system of equations

$$\begin{aligned} x + y + 2z &= 7 \\ x + 2y + 3z &= 9 \\ x + 2y + 4z &= 10 \end{aligned}$$

Answer. x = ___________ , y = ___________ , z = ___________

Problem 2. Calculate the trace of A^{-1}, where $A =$

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 0 & 2 & 1\,. \end{matrix}$$

Answer. Trace of A^{-1} = __________

Problem 3. Find a 2×2 matrix A with eigenvalues 1 and 2 and corresponding eigenvectors (3, 1) and (2, 1).

Answer. A =

Problem 4. Diagonalize the matrix A =

$$\begin{matrix} 4 & -3 \\ 1 & 0 \end{matrix}$$

That is, find matrices P and D such that $A = P\,D\,P^{-1}$, where D is diagonal.

You must put the following answers in the designated spaces:

(1) Eigenvalues of A in increasing order: __________ and __________

(2) Eigenvectors of A in corresponding order: ____________ and ___________

(3) Diagonal matrix D =

(4) Matrix P =

Extra page for Problem 4.

Problem 5. Find the general solution of the differential equation

$$d^4y/dx^4 \; + \; 4\, d^3y/dx^3 \; + \; 4\, d^2y/dx^2 \; = \; 0\,.$$

Answer. $y(x) =$ ____

Problem 6. Find the function $y(x)$ which satisfies the differential equation

$$x^2 y'' - 5x y' + 8 y = 0$$

and the initial conditions $y(2) = 32$ and $y'(2) = 0$.

Answer. $y(x) =$ ____

Problem 7. The differential equation

$$(x^2 + 1)\, y'' + x\, y' + y = 0$$

has solutions in the form of power series

$$y = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n + \ldots .$$

Find the recursion formula for the coefficients.

Answer. Recursion formula is ____________________________________

Problem 8. The differential equation

$$3xy'' + y' - y = 0$$

has solutions of the form

$$y = x^r (c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n + \dots) .$$

Find the two possible values of r .

Answer. $r =$ __________ and __________

Problem 9. Consider the vector field

$$V = (x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}) / (x^2 + y^2 + z^2)\,.$$

Compute the outward flux of V through the boundary of the region R between the spheres of radius a and b $(a < b)$ centered at the origin.

Answer. Outward flux of V = ____________________________________

Problem 10. Consider the vector field

$$V = (\cos z + y \cos x)\,\mathbf{i} + (\sin x)\,\mathbf{j} + w(x, y, z)\,\mathbf{k}\,,$$

and suppose that the line integral of V along any path in 3-space depends only on the endpoints of the path.

Find all functions $w(x, y, z)$ which make this possible.

Answer. $w(x, y, z) =$ ____

Problem 11. Let C be the ellipse in the xy-plane given by the equation

$$x^2/4 + y^2/16 = 1 ,$$

oriented counterclockwise. Find the value of the integral $\int_C y\,dx - x\,dy$.

Answer. $\int_C y\,dx - x\,dy =$ ______________________________

Problem 12. Let

$$V = y\,\mathbf{i} + yz^3\,\mathbf{j} + xyz\,\mathbf{k}.$$

Compute the outward flux of the curl of V through the portion of the surface

$$4 - z = x^2 + y^2$$

which lies above the xy-plane.

Answer. Outward flux of the curl of V = ____________________________

END OF EXAM