| Problems | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Score | | | | | | | | | | | | |

Total:

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PLEASE DO NOT WRITE ABOVE THIS LINE

***UNIVERSITY OF PENNSYLVANIA***
***DEPARTMENT OF MATHEMATICS***

**MATH 240 FINAL EXAM**

**Wednesday December 17, 2008**

Your name (printed) ____________________________________________________

Signature ____________________________________________________________

Professor (circle one): Michael Temkin Peter Quast Herman Gluck

Recitation TA _____________Recitation Day __________Recitation Hour _________

**INSTRUCTIONS.**

There are 12 problems on this exam.

Problems 1 - 10 are worth 8 points each, and we will select the best 8 of these.
***There is no partial credit for these questions.***
To get full credit, you must get the right answer and your written supporting work must be understandable by us (the graders) and be sufficient in our opinion to fully derive your answer. Even if you can do part of the problem in your head, you must show all the work.

Problems 11 and 12 are worth 18 points each. You must show all details, and put your answers in the spaces provided.
***Partial credit is available for these two questions.***

No books, notes, calculators or computers during the exam...except that you may use one 8 1/2 by 11 inch sheet of paper with information written or typed on both sides.

This exam is conducted under the Code of Academic Integrity of the University of Pennsylvania.

Please write **dark** and large, and write your name on each sheet, in the space provided.

Good luck!

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**Problem 1.** Solve the system of linear equations

$$\begin{aligned} 2x + 3y + z &= 1 \\ 2y + 3z &= -1 \\ x + 3y - z &= -4 \end{aligned}$$

and find the value of $y$ .

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) $-2$ (b) $-1$ (c) 0 (d) 1 (e) 2

**Problem 2.** Find $\det(A^{-1} B A)$, where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -5 & -2 & 0 \\ 3 & -5 & 3 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 2 & 3 & 0 \\ 4 & 1 & 2 \\ -1 & -2 & 0 \end{pmatrix}$$

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) –2 (b) –1 (c) 0 (d) 1 (e) 2

**Problem 3.** Let M be the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ -1 & -2 & -2 \end{pmatrix}$$

Compute the sum of the elements in the last **column** of $M^{-1}$.

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) $-2$ (b) $-1$ (c) 0 (d) 1 (e) 2

**Problem 4.** Start with the vector field $V = x^3 y\,\mathbf{i} + y^3 z\,\mathbf{j} + z^3 x\,\mathbf{k}$, compute the divergence of the gradient of the divergence of $V$, and evaluate the answer at the point $(1, 0, 1)$.

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) $-12$ (b) $-6$ (c) $0$ (d) $6$ (e) $12$

**Problem 5.** Evaluate the line integral $\int_C -y\,dx + x\,dy$,
where C is the circle $(x-2)^2 + (y-3)^2 = 4$ oriented counterclockwise.

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) $-8\pi$ (b) $-4\pi$ (c) $0$ (d) $4\pi$ (e) $8\pi$

**Problem 6.** Let S be the surface of the ellipsoid $4x^2 + 4y^2 + 9z^2 = 4$ with $z \geq 0$, oriented by the outward pointing unit normal vector field **n** .
Let V be the vector field $V = y\,\mathbf{i} + x^2\,\mathbf{j} + \sin(xyz)\,\mathbf{k}$ .
Compute the flux of the curl of V through S .

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) $-2\pi$ (b) $-\pi$ (c) $0$ (d) $\pi$ (e) $2\pi$

**Problem 7.** Let C be the rectangular box consisting of all points (x, y, z) with $0 \le x \le 1$, $0 \le y \le 2$ and $0 \le z \le 3$. Let V be the vector field

$$V = (2x + x^3)\,\mathbf{i} + (\sin y - 2yz - 3x^2y)\,\mathbf{j} + (z^2 - z\cos y)\,\mathbf{k}.$$

Find the flux of the vector field V through the boundary of the box C.

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) –24 (b) –12 (c) 0 (d) 12 (e) 24

**Problem 8.** Suppose that the function $y(x)$ satisfies the differential equation $y'' + y' - 6y = 6$ with initial values $y(0) = 1$ and $y'(0) = -1$.
Find the value of $y(-1)$.

CIRCLE ONE OF THE FOLLOWING ANSWERS.

(a) $e^{-3} - e^{2} - 1$ (b) $e^{-3} + e^{2} - 1$ (c) $e^{3} - e^{-2} - 1$ (d) $e^{3} + e^{-2} - 1$ (e) $e^{3} + e^{-2} + 1$

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**Problem 9.** Solve the differential equation $x^2 y'' + 2xy' = 0$ with $y(1) = 1$ and $y'(1) = 1$.

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**You must put your answer here:**

$y(x) =$ ________________________________.

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**Problem 10.** Solve the following system of first order linear differential equations:

$$dx/dt = x + 3y$$
$$dy/dt = 3x + y$$

with initial conditions $x(0) = 0$ and $y(0) = 1$ .

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**You must put your answer here:**

$x(t) =$ ____________________________________.

$y(t) =$ ____________________________________.

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**Problem 11.** Diagonalize the matrix A =

$$\begin{matrix} 2 & -1 \\ 2 & 5 \end{matrix}$$

That is, find matrices P and D such that $A = P D P^{-1}$, where D is diagonal.

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**You must put the following answers here:**

(1) Eigenvalues of A (smaller one first) are __________ and __________ .

(2) The corresponding eigenvectors of A (in the same order) are

______________ and _________________

(3) The diagonal matrix D =

(4) The matrix P =

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**Extra sheet for Problem 11.**

**Problem 12.** Solve

$$y'' - x^2 y = 0 \quad , \; y(0) = 1 \, , \; y'(0) = 1$$

by the method of power series, where $y = c_0 + c_1 x + c_2 x^2 + \ \dots + c_n x^n + \dots$, giving the recursion formula for the coefficients, and the specific coefficient $c_4$ of $x^4$ in the power series for $y$ .

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**You must put your answers here.**

(1) The recursion formula for the coefficients of the power series is:

____________________________________

(2) The coefficient of $x^4$ is ____________________

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**Extra sheet for Problem 12.**

**END OF EXAM**