MATHEMATICS 240 **J. KRIEGER, N. RIMMER, S. SHATZ**
FINAL EXAMINATION **DECEMBER 12, 2007** **9:00 AM**

Answer all questions by circling the **entire** statement you deem correct in each question. No books, tables, notes, calculators, computers or cell phones allowed. You may bring one 8.5" x 11" sheet of paper bearing any handwritten material you deem necessary and you may use both sides of this paper. **No partial credit.** Use the backs of your exam pages for scratch work and calculations.

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(Your name—please print) **(Your PENN ID number)**

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(Your signature) **(Your instructor's name (print))**

PLEASE DO NOT WRITE BELOW THIS LINE

PROBLEM	SCORE	PROBLEM	SCORE	PROBLEM	SCORE
I		VI		XI	
II		VII		XII	
III		VIII		XIII	
IV		IX		XIV	
V		X		XV	

TOTAL------------------

(Your name)

I) For the differential equation

$$y'' - x^2y' + xy = 0$$

and the solution, y(x), determined by the initial conditions y(0) = 0 and y′(0) = 1, when we write y(x) in a power series, the coefficient of x^4 in this series is:

a) -1/24 b) 1/2 c) -1/6 d) 2/3 e) 0

II) Given the vectors: (1, t, -1); (0, 1, -2); (1, (s + 2), -s), find the condition on t and s so that these vectors are linearly dependent.

a) s = 2t + 5 b) s = 2t - 5 c) t - s = 5

d) t + s = 5 e) none of these

III) An iron bead is constrained to move along the x-axis and is acted on by a varying magnetic field. The differential equation describing its motion is:

$$x'' + (1/2)\, x' + 2x = \cos(t).$$

Suppose x(0) = 4/5 and x′(0) = 2/5, then x(3π/2) equals:

a) 4/5 b) 3/5 c) -2/5 d) 2/5 e) -4/5

IV) Suppose A and B are 3 x 3 matrices and det A = x ≠ 0 while det B = y. Let C be the matrix $(2A)^{-1}B$, then det C is:

a) y/8x b) 2xy c) -2y/x d) 2y/x e) 8y/x

(Your name)

V) Given a 4 x 4 matrix, M, we form the augmented 4 x 8 matrix Y = (M | I) and row reduce Y to row reduced echelon form. When this is done we get the new matrix

$$\tilde{Y} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

If $X = (x_1, x_2, x_3, x_4)$ and $b = (b_1, b_2, b_3, b_4)$, where the x_i are unknowns and the b_i are given numbers and each of X and b is viewed as a column vector, then we consider the matrix equation MX = b and for it we can say:

a) For each b there is a unique solution, but it may not be Cb.

b) There are vectors b for which there is no solution.

c) For each given b there are always many solutions.

d) For each b there is a unique solution; it is Cb.

e) Need more information about M and b to decide among the above choices.

VI) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ in which C is the curve $\mathbf{r}(t) = (t, t^2, t^3)$ for $0 \le t \le 1$ and $\mathbf{F}$ is the vector field $(e^y, xe^y, (z + 1)e^z)$.

a) 1/2 b) 2e c) 0 d) e e) e/2

VII) Recall that for a matrix its trace is the sum of its eigenvalues and its determinant is the product of its eigenvalues. Which of the following is **false**?

a) There is a symmetric 3x3 matrix with an eigenvalue 2, trace 5 and det = 4.

b) For any 3 x 3 matrix, A, the matrix $A(A^T)$ is always diagonalizable.

c) Symmetric matrices always have an orthonormal basis of eigenvectors.

d) There is a symmetric 3x3 matrix with an eigenvalue 2, trace 5 and det = 6.

e) None of the above.

(Your name)

VIII) Consider the region Ω whose boundary consists of **two** curves: $x^2 + y^2 = 4$ and $x^2 + y^2 = ½$ oriented and labeled as shown in the sketch below. Write C for the boundary of Ω and let **F** be the vector field

$$\mathbf{F}(x, y) = (-y/[(x - 1)^2 + y^2], (x - 1)/[(x - 1)^2 + y^2]).$$

Then $\int_C \mathbf{F} \cdot d\mathbf{r}$ equals:

a) 2π b) -4π c) 4π d) 0 e) -2π

IX) When we solve the differential equation

$$x^2(2 + x)y'' + 2xy' - 3y = 0$$

by the Frobenius method, the exponents, r, in the solution $y = x^r \sum c_i x^i$ are:

a) $r = \pm 2$ b) $r = -1 \pm \sqrt{3}$ c) $r = \pm\sqrt{3}$ d) $r = \pm\sqrt{(3/2)}$ e) $r = \pm 1$

X) Evaluate the line integral $\int_C (y + e^{\sqrt{x}})dx + (2x - \cos(y^2))dy$, where C is the boundary of the region enclosed between the curves $y = x^2$ and $x = y^2$ and C is oriented counter-clockwise.

a) $1/3$ b) 3 c) 0 d) $\pi/3$ e) π

(Your name)

XI) Suppose A is the 3 x 3 matrix:

1	1	0
0	1	0
0	-1	2

Then

a) A has eigenvalues 1, 1, 2 and is not diagonalizable.

b) A has eigenvalues 0, 1, 2 and is not diagonalizable.

c) A has eigenvalues 1, 1, 2 and is diagonalizable.

d) A has eigenvalues 0, 1, 2 and is diagonalizable.

e) None of the above.

XII) For the initial value problem:

$$y'' - 3y' + 2y = U(t-1)\ ;\ y(0) = 0,\ y'(0) = 1,$$

in which U(t) is the function that is 0 when $t < 0$ and 1 when $t \geq 0$, the Laplace transform of y, namely L(y)(s), is given by

a) $1/(s^2 - 3s + 2) + e^{-s}/(s(s^2 - 3s + 2))$

b) $1/(s^2 - 3s + 2) - e^{-s}/(s(s^2 - 3s + 2))$

c) $1/(s^2 - 3s + 2)$

d) $1/s(s^2 - 3s + 2)$

e) Cannot be determined

(Your name)

XIII) Consider the surface, S, formed by the upper half of the ellipsoid

$$x^2 + y^2 + 6z^2 = 1,$$

and write C for the circle $x^2 + y^2 = 1$ where S cuts the xy-plane. We use the outer normal (upward pointing) to orient S so that C is traversed counter-clockwise in the xy-plane (when viewed from above). Let **F** be the vector field

$$(\sin(xz) + \sqrt{z}\,,\ (3 + z)x - e^y,\ x^2 - y^3 - z^5)$$

Compute the surface integral $\iint_S$ **curl F · n** dS:

a) $-\pi$ b) π c) -3π d) 3π e) none of these

XIV) We solve the differential equation $y'' + (t - (t^3/6))y = 0$ by a series $\sum a_i t^i$ and assume the initial conditions $y(0) = y'(0) = 1$.
Then the expression

$$(a_2)^3 + (a_3)^2 - a_4$$

equals:

a) -1/3 b) 1/9 c) 1/12 d) -1/12 e) 1/6

XV) Write S for the part of the surface $z = x^2 + y^2$ over the disc $x^2 + y^2 \le 1$ in the plane. We orient S so that its normal, **n**, points downward. If the vector field, **F**, is given by: $\mathbf{F}(x, y, z) = (e^y + x,\ e^{\sin(z)} + \sin(x),\ -z + xy)$, then the surface integral $\iint_S$ **F · n** dS equals:

a) 2π b) 4π c) π d) $\pi/2$ e) None of these.

(END OF THE EXAM)