Math 240 Final Exam
Spring 2012

1. Solve the equations for x.

$$\begin{array}{rcrcrcr} 2x & + & 3y & + & 2z & = & 1 \\ x & + & 0y & + & 3z & = & 2 \\ 2x & + & 2y & + & 3z & = & 3 \end{array}$$

Hint:

$$\text{If } A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 3 \end{bmatrix} \text{ then } A^{-1} = \begin{bmatrix} -6 & -5 & 9 \\ 3 & 2 & -4 \\ 2 & 2 & -3 \end{bmatrix}.$$

a) $x = -1$ b) $x = 0$ c) $x = 2$ d) $x = 5$ e) $x = 7$ f) $x = 11$ g) none of the above

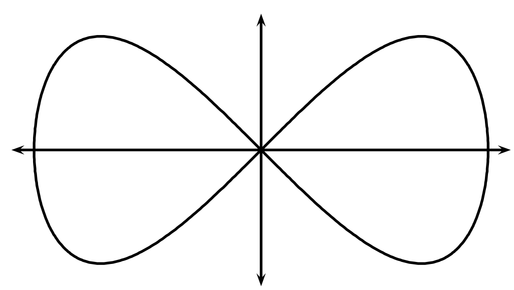
Answer: (f). To find the solution vector, we need only apply the inverse:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 & -5 & 9 \\ 3 & 2 & -4 \\ 2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}.$$

This answer can easily be verified by substituting back into the original system.

2. The **lemniscate of Gerono** is parametrized by the formulas

$$x(t) = \cos t,$$
$$y(t) = \sin t \cos t.$$



Compute the area of the right-hand lobe (corresponding to the range of parameters $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$). Hint: Use Green's Theorem and the differential $-y\,dx$. Near the end you'll likely need to use a u-substitution.

a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$ e) $\frac{5}{6}$ f) 1 g) none of the above

Answer: (d). For this specific differential, we have $P = -y$ and $Q = 0$, so $Q_x - P_y = 1$. Thus by Green's Theorem we have

$$\iint_A 1 \frac{d}{A} = \int_{\partial A} -y dx,$$

that is, the area of A is equal to the line integral of $-ydx$ along the boundary of A. Writing the line integral using the given parametrization, we have

$$\int_{\partial A} -y dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\sin t \cos t (-\sin t) dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t \cos t dt.$$

Substituting $u = \sin t$ gives

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t \cos t dt = \int_{-1}^{1} u^2 du = \frac{2}{3}.$$

3. Calculate the outward flux of $\vec{F}$ across S if $\vec{F}(x,y,z) = 3xy^2\vec{\imath} + xe^z\vec{\jmath} + z^3\vec{k}$ and S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

a) 0 b) $-\frac{\pi}{4}$ c) $\frac{11\pi}{8}$ d) 3π e) $\frac{9\pi}{5}$ f) $\frac{9\pi}{2}$ g) none of the above

Answer: (f). We use the divergence theorem. First we compute the divergence of $\vec{F}$:

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(3xy^2) + \frac{\partial}{\partial y}(xe^z) + \frac{\partial}{\partial z}(z^3) = 3(y^2 + z^2).$$

If C is the interior of the cylinder that is described in the problem and ∂C is the surface bounding it, then

$$\iint_{\partial C} \vec{F} \cdot \vec{n} dS = \iiint_C 3(y^2 + z^2) dV.$$

We can evaluate the integral in cylindrical coordinates (r, θ, x), where (r, θ) represent the polar coordinates in the yz-plane.

$$\iint_{\partial C} \vec{F} \cdot \vec{n} dS = \iiint_C 3(y^2 + z^2) dV = \int_{-1}^{2} \int_0^{2\pi} \int_0^1 3r^2 r dr d\theta dx = \frac{9\pi}{2}.$$

4. Compute the outward flux of $\nabla \times \vec{F}$ through the surface of the ellipsoid $2x^2 + 2y^2 + z^2 = 8$ lying above the plane $z = 0$, where

$$\vec{F} = (3x - y)\vec{\imath} + (x + 3y)\vec{\jmath} + (1 + x^2 + y^2 + z^2)\vec{k}.$$

a) 0 b) 2π c) 3π d) 8π e) 12π f) 16π g) none of the above

Answer: (d). We use Stokes' theorem. If S is the top half of the ellipsoid, then its boundary will be a curve C obtained by setting $z = 0$. On the circle, $2x^2 + 2y^2 = 8$, meaning that C is the circle of radius 2 centered at the origin and lying in the xy-plane. Since the outward flux points upwards in the z-direction, the correct orientation of C is counterclockwise. We conclude

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dS = \oint_C (3x - y)dx + (x + 3y)dy + (1 + x^2 + y^2 + z^2)dz.$$

We can parametrize C by $\vec{r}(t) = \langle 2\cos t, 2\sin t, 0\rangle$ for $t \in [0, 2\pi]$. We will get

$$\begin{aligned}\oint_C (3x - y)dx + &(x + 3y)dy + (1 + x^2 + y^2 + z^2)dz \\ &= \int_0^{2\pi} \left[(6\cos t - 2\sin t)(-2\sin t) + (2\cos t + 6\sin t)(2\cos(t)) + (1 + 4\cos^2 t + 4\sin^2 t)(0)\right] dt. \\ &= \int_0^{2\pi} 4\left[\sin^2 t + \cos^2 t\right] dt = 8\pi.\end{aligned}$$

5. Find a 2×2 real matrix A that has

$$\text{an eigenvalue } \lambda_1 = 1 \text{ with eigenvector } \vec{E}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and an eigenvalue } \lambda_2 = -1 \text{ with eigenvector } \vec{E}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Then compute the determinant of $A^{10} + A$ and write your answer in the box below.

a) $A = \begin{bmatrix} -\frac{5}{3} & \frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix}$ b) $A = \begin{bmatrix} -\frac{4}{3} & \frac{5}{3} \\ -\frac{5}{3} & \frac{4}{3} \end{bmatrix}$ c) $A = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix}$ d) $A = \begin{bmatrix} \frac{4}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{4}{3} \end{bmatrix}$

e) $A = \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ \frac{4}{3} & -\frac{5}{3} \end{bmatrix}$ f) $A = \begin{bmatrix} \frac{4}{3} & -\frac{5}{3} \\ \frac{5}{3} & -\frac{4}{3} \end{bmatrix}$ g) none of the above

$$\det(A^{10} + A) = \boxed{}$$

Answer: (a). If such a matrix A exists, it should be diagonalizable, meaning $D = P^{-1}AP$ when

$$D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ when } P = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

We can find A by the formula $A = PDP^{-1}$. We have

$$P^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}, \qquad DP^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}, \qquad PDP^{-1} = \begin{bmatrix} -\frac{5}{3} & \frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix}.$$

We can compute A^{10} using the diagonalization: $A^{10} = PD^{10}P^{-1}$. The tenth power of D is the identity matrix, so $A^{10} = PIP^{-1} = PP^{-1} = I$. We conclude

$$\det(A^{10} + A) = \det(A + I) = \det \begin{bmatrix} -\frac{2}{3} & \frac{4}{3} \\ -\frac{4}{3} & \frac{8}{3} \end{bmatrix} = 0.$$

Alternately, $\det(A^{10} + A) = \det(D^{10} + D) = 0$ since $D + I$ has a row which is identically zero.

6. Identify all possible eigenvalues of an $n \times n$ matrix A if A which satisfies the following matrix equation:

$$A - 2I = -A^2.$$

Must A be invertible? Record your answer in the box below and provide justification for your answer.

a) $\lambda = 0, 1$ b) $\lambda = 0, 2$ c) $\lambda = 0, 1, -2$ d) $\lambda = 1, -2$ e) $\lambda = 0, 1, -3$ f) $\lambda = 1, -3$ g) none of the above

$$\text{Is } A \text{ invertible? } \boxed{}$$

Answer: (d). Suppose $\vec{E}$ is an eigenvector of A with eigenvalue λ (recall that every eigenvalue has at least one eigenvector). We know that $A\vec{E} = \lambda\vec{E}$, so $(A - 2I)\vec{E} = (\lambda - 2)\vec{E}$. Likewise, $-A^2\vec{E} = -A(\lambda\vec{E}) = -\lambda^2\vec{E}$. We conclude that $(\lambda - 2)\vec{E} = -\lambda^2\vec{E}$ when the relationship $A - 2I = -A^2$ holds. Since $\vec{E}$ is never zero, we have $\lambda - 2 = -\lambda^2$. Solving for λ gives $\lambda = 1, -2$. These are the only possible eigenvalues–we haven't shown that they *must* be eigenvalues (take $A = I$, for example; it satisfies the equation but has not eigenvalue equal to -2), but that no other possible eigenvalues could occur. In particular, A must be invertible because 0 cannot be an eigenvalue. Alternately, notice that our formula implies that

$$I = \frac{1}{2}(A + A^2) = A\left(\frac{1}{2}I + \frac{1}{2}A\right).$$

Not only must A be invertible, but its inverse equals $\frac{1}{2}I + \frac{1}{2}A$.

7. Solve the differential equation

$$9x^2y'' + 2y = 0$$

on the interval $(0, \infty)$ subject to the initial conditions $y(1) = 1$ and $y'(1) = \frac{4}{3}$.

a) $y = 2x^{\frac{2}{3}} - 3x^{\frac{1}{3}}$ b) $y = 3x^{\frac{2}{3}} - 2x^{\frac{1}{3}}$ c) $y = 3x^{\frac{3}{2}} - 3x^3$ d) $y = 3x^{\frac{3}{2}} - 2x^3$
e) $y = 2x^2 - 3x$ f) $y = 3x^2 - 2x$ g) none of the above

Answer: (b). This is a Cauchy-Euler equation. The auxiliary equation is $9m(m-1) + 2 = 0$, which factors as $(3m - 2)(3m-1) = 0$. Thus the general solution will have the form $y = c_1x^{\frac{2}{3}} + c_2x^{\frac{1}{3}}$. Solving the system of equations

$$\begin{aligned} c_1 + c_2 &= 1 \\ \frac{2}{3}c_2 + \frac{1}{3}c_2 &= \frac{4}{3} \end{aligned}$$

gives $c_1 = 3$ and $c_2 = -2$.

8. Let $\vec{\omega} := \langle 1, 2, 3\rangle$, and let $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$. Now consider the differential equation

$$\frac{d}{dt}\vec{r} = \vec{\omega} \times \vec{r}.$$

Select the answer which correctly expresses this system of equations in matrix notation when

$$\vec{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}.$$

Do not solve the system.

a) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}\vec{X}$ b) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}\vec{X}$ c) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}\vec{X}$

d) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}\vec{X}$ e) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}\vec{X}$ f) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}\vec{X}$

g) none of the above

Answer: (c). First we simply expand the formula for the cross product:

$$\begin{aligned} \langle 1,2,3\rangle \times \langle x,y,z\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ x & y & z \end{vmatrix} &= \begin{vmatrix} 2 & 3 \\ y & z \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 3 \\ x & z \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 2 \\ x & y \end{vmatrix}\vec{k} \\ &= (2x - 3y)\vec{i} + (3x - z)\vec{j} + (y - 2x)\vec{k}. \end{aligned}$$

We write this vector as a column vector and discover that

$$\begin{bmatrix} 2x - 3y \\ 3x - z \\ y - 2x \end{bmatrix} \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

$$\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}\vec{X}.$$

9. Select the answer below which corresponds to the first few terms in a power series solution of the differential equation

$$x^2y'' + (x^2 - x)y' + y = 0.$$

Will there be a second, linearly independent series solution for this equation? Explain your answer.

a) $y = x^{\frac{1}{2}} + \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{6}x^{\frac{5}{2}} + \frac{1}{12}x^{\frac{7}{2}} + \cdots$ b) $y = -x^{\frac{1}{2}} + \frac{1}{6}x^{\frac{3}{2}} - \frac{1}{12}x^{\frac{5}{2}} + \frac{1}{20}x^{\frac{7}{2}} + \cdots$ c) $y = x^{\frac{1}{2}} + x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{5}{2}} - \frac{1}{9}x^{\frac{7}{2}} + \cdots$

d) $y = x - x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \cdots$ e) $y = x - x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^4 + \cdots$ f) $y = x + x^2 + \frac{1}{2}x^3 + \frac{1}{9}x^4 + \cdots$

g) none of the above

Answer: (e). The point $x = 0$ is a regular singular point of this ODE. We use the method of Frobenius to find a solution

$$y = \sum_{n=0}^{\infty} c_n x^{n+r} \qquad xy' = \sum_{n=0}^{\infty} (n+r)c_n x^{n+r} \qquad x^2y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1)c_n x^{n+r}$$

We substitute in and get

$$\begin{aligned} x^2y'' + (x^2 - x)y' + y &= \sum_{n=0}^{\infty}(n+r)(n+r-1)c_n x^{n+r} + \sum_{n=0}^{\infty}(n+r)c_n x^{n+r+1} - \sum_{n=0}^{\infty}(n+r)c_n x^{n+r} + \sum_{n=0}^{\infty} c_n x^{n+r} = 0 \\ &= \sum_{n=0}^{\infty}(n+r)(n+r-1)c_n x^{n+r} + \sum_{n=1}^{\infty}(n-1+r)c_{n-1} x^{n+r} - \sum_{n=0}^{\infty}(n+r)c_n x^{n+r} + \sum_{n=0}^{\infty} c_n x^{n+r} \\ &= [r(r-1)c_0 - rc_0 + c_0] + \sum_{n=1}^{\infty}\left[(n+r)(n+r-1)c_n + (n-1+r)c_{n-1} - (n+r)c_n + c_n\right] x^{n+r} \end{aligned}$$

We simplify and conclude that

$$(r-1)^2 c_0 = 0 \text{ and } (n+r-1)^2 c_n + (n+r-1)c_{n-1} = 0 \text{ when } n \geq 1.$$

The indicial roots are both $r = 1$, so the method of Frobenius says that only one such series solution will exist (the other will involve a logarithm). We have

$$c_1 = -\frac{1}{1+1-1}c_0 = -c_0 \qquad c_2 = -\frac{1}{2+1-1}c_1 = \frac{c_0}{2} \qquad c_3 = -\frac{1}{3+1-1}c_2 = -\frac{c_0}{6}.$$

In fact, we can see that there is a pattern: $c_n = \frac{(-1)^n}{n!}$. Our full series will be

$$y = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{n+1}.$$

We can even identify this as a Taylor series: $y = xe^{-x}$.

10. Let y be a function satisfying $y(0) = y'(0) = y''(0) = 0$ which is a solution of the ODE

$$y''' - 4y'' + 4y' = 4.$$

Compute $y(1)$.

a) $y(1) = -5$ b) $y(1) = 4$ c) $y(1) = -3$ d) $y(1) = 2$ e) $y(1) = -1$ f) $y(1) = 0$ g) none of the above

Answer: (d). The complementary solution will have the form $y_c = c_1e^{2x} + c_2xe^{2x} + c_3$. The method of undetermined coefficients says that we should find a particular solution in the form $y_p = Ax$ (we multiply by x because the complementary solution includes constants). That method gives a solution when $A = 1$. Thus the solution is of the form

$y = c_1e^{2x} + c_2xe^{2x} + c_3 + x$. To solve the IVP we differentiate this solution $0, 1$, and 2 times before setting $x = 0$ in each case:

$$\begin{aligned} c_1 + c_3 &= 0 \\ 2c_1 + c_2 + 1 &= 0 \\ 4c_1 + 4c_2 &= 0 \end{aligned}$$

The solution of this system is $c_1 = -1$, $c_2 = c_3 = 1$, so we have

$$y = -e^{2x} + xe^{2x} + 1 + x.$$

11. Solve the following system of differential equations subject to the initial conditions $y_1(0) = 1$ and $y_2(0) = 3$. **Clearly state your solution.** What is $y_1(1)$?

$$\begin{aligned} \frac{dy_1}{dx} &= 3y_1 - y_2 \\ \frac{dy_2}{dx} &= \ \ y_1 + y_2 \end{aligned}$$

a) $y_1(1) = 2e$

b) $y_1(1) = 2e - 1$

c) $y_1(1) = 3$

d) $y_1(1) = 5e^2$

e) $y_1(1) = 7e$

f) $y_1(1) = -e^2$

g) none of the above

Answer: (f). We find that the characteristic equation of the matrix

$$\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$$

is $(3 - \lambda)(1 - \lambda) + 1 = 0$, or $\lambda^2 - 4\lambda + 4 = 0$. This has a double root $\lambda = 2$. There will be only one eigenvector, for example,

$$\vec{E} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For the second solution, we need a generalized eigenvector $\vec{E}_2$ such that

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \vec{E}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

One possible solution (out of infinitely many possibilities) is

$$\vec{E}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

We conclude that the general solution of the system has the form

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = C_1e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \left(te^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{2t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right).$$

Now solve

$$C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

We find that $C_1 = 3$ and $C_2 = -2$. Consequently

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} e^{2t} - 2te^{2t} \\ 3e^{2t} - 2te^{2t} \end{bmatrix}.$$

Plugging in $t = 1$ gives the answer.

12. Find a solution to the initial value problem $y'' - 2xy' - 4y = 0$ subject to the initial conditions $y(0) = 0$ and $y'(0) = 1$ which takes the form of a power series centered at the origin. What is the coefficient in front of x^5 in the series?

a) -1 b) 0 c) $\frac{1}{2}$ d) 1 e) 2 f) 6 g) none of the above

Answer: (c). The point $x = 0$ is ordinary, so we make a standard power series solution:

$$y = \sum_{n=0}^{\infty} c_n x^n \qquad y' = \sum_{n=0}^{\infty} n c_n x^{n-1} \qquad y'' = \sum_{n=0}^{\infty} n(n-1) c_n x^{n-2}.$$

We substitute these into the equation and get

$$\sum_{n=0}^{\infty} n(n-1) c_n x^{n-2} - 2x \sum_{n=0}^{\infty} n c_n x^{n-1} - 4 \sum_{n=0}^{\infty} c_n x^n = 0.$$

Next we can combine the last two sums:

$$\sum_{n=0}^{\infty} n(n-1) c_n x^{n-2} + \sum_{n=0}^{\infty} (-2n-4) c_n x^n = 0.$$

After that we shift the index of summation in the first sum:

$$\sum_{n=-2}^{\infty} (n+2)(n+1) c_{n+2} x^n + \sum_{n=0}^{\infty} (-2n-4) c_n x^n = 0.$$

Because $(n+2)(n+1) = 0$ both when $n = -2$ and $n = -1$, we can drop these two terms from the first sum and combine again:

$$\sum_{n=0}^{\infty} \left[(n+2)(n+1) c_{n+2} - 2(n+2) c_n\right] x^n.$$

Our recurrence formula is found by setting these coefficients equal to zero:

$$(n+2)(n+1) c_{n+2} - 2(n+2) c_n = 0 \Rightarrow c_{n+2} = \frac{2c_n}{n+1}.$$

The initial conditions give $c_0 = 0$ and $c_1 = 1$. Computing the rest by means of the formula gives

$$c_2 = \frac{2c_0}{1} = 0 \qquad c_3 = \frac{2c_1}{2} = 1 \qquad c_4 = 0 \qquad c_5 = \frac{2c_3}{4} = \frac{1}{2}.$$

13. Circle "T" for true or "F" for false in the space provided to the left of the following statements. You **DO NOT** need to justify your answer for full credit.

(T F) Every 2×2 diagonalizable matrix with repeated eigenvalue is a diagonal matrix.
True: $A = PDP^{-1}$ and D is a multiple of the identity, so A must also be.

(T F) There is a vector field $\vec{F}$ such that $\nabla \times \vec{F} = \langle x, y, z \rangle$.
False: The divergence of the curl is always equal to zero, but the divergence of $\langle x, y, z \rangle$ equals 3.

(T F) If $\det(A) = 0$, then the system $A\vec{X} = 0$ has infinitely many solutions.
True: $\det(A) = 0$ means that A has an eigenvector $\vec{E}$ which has eigenvalue zero. Any multiple of $\vec{E}$ will solve the system $A\vec{X} = 0$.

(T F) If y_1 and y_2 are solutions to a non-homogeneous linear differential equation, then $y_1 + y_2$ is also a solution.
False: Try the differential equation $y' = 1$ and take $y_1 = y_2 = x$.

(T F) If A and B are square matrixes such that $AB^2 = I$, then B is invertible.
True: $\det(AB^2) = (\det A)(\det B)(\det B) = \det I = 1$. In particular, $\det A$ and $\det B$ must both be nonzero, meaning they're both invertible.