Math 240 Final Exam
Spring 2012

Name:

Instructor Name:

TA Name:

Recitation Number or Day/Time:

Please *turn off and put away all electronic devices*. You may use both sides of a 8.5″ × 11″ sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. **Show all work** on multiple choice questions: one point will be given for clearly circling the correct answer, and up to four points will be giving for the supporting work. Please **clearly mark** a multiple choice option for each problem. Remember to put your name at the top of this page. Good luck.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

____

Your signature

For Official Use (do not write below this line)

Question Number	Points Possible	Your Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
13	5	
Total	65	

1. Solve the equations for x.

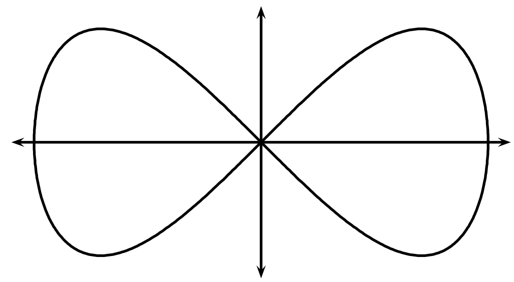
$$\begin{array}{rcrcrcr} 2x & + & 3y & + & 2z & = & 1 \\ x & + & 0y & + & 3z & = & 2 \\ 2x & + & 2y & + & 3z & = & 3 \end{array}$$

Hint:

$$\text{If } A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 3 \end{bmatrix} \text{ then } A^{-1} = \begin{bmatrix} -6 & -5 & 9 \\ 3 & 2 & -4 \\ 2 & 2 & -3 \end{bmatrix}.$$

a) $x = -1$ b) $x = 0$ c) $x = 2$ d) $x = 5$ e) $x = 7$ f) $x = 11$ g) none of the above

2. The **lemniscate of Gerono** is parametrized by the formulas



$$x(t) = \cos t,$$
$$y(t) = \sin t \cos t.$$

Compute the area of the right-hand lobe (corresponding to the range of parameters $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$). Hint: Use Green's Theorem and the differential $-y\,dx$. Near the end you'll likely need to use a u-substitution.

a) $\dfrac{1}{6}$ b) $\dfrac{1}{3}$ c) $\dfrac{1}{2}$ d) $\dfrac{2}{3}$ e) $\dfrac{5}{6}$ f) 1 g) none of the above

3. Calculate the outward flux of $\vec{F}$ across S if $\vec{F}(x,y,z) = 3xy^2\vec{\imath} + xe^z\vec{\jmath} + z^3\vec{k}$ and S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

a) 0 b) $-\dfrac{\pi}{4}$ c) $\dfrac{11\pi}{8}$ d) 3π e) $\dfrac{9\pi}{5}$ f) $\dfrac{9\pi}{2}$ g) none of the above

4. Compute the outward flux of $\nabla \times \vec{F}$ through the surface of the ellipsoid $2x^2 + 2y^2 + z^2 = 8$ lying above the plane $z = 0$, where

$$\vec{F} = (3x - y)\vec{\imath} + (x + 3y)\vec{\jmath} + (1 + x^2 + y^2 + z^2)\vec{k}.$$

a) 0 b) 2π c) 3π d) 8π e) 12π f) 16π g) none of the above

5. Find a 2×2 real matrix A that has

$$\text{an eigenvalue } \lambda_1 = 1 \text{ with eigenvector } \vec{E}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and an eigenvalue } \lambda_2 = -1 \text{ with eigenvector } \vec{E}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Then compute the determinant of $A^{10} + A$ and write your answer in the box below.

a) $A = \begin{bmatrix} -\frac{5}{3} & \frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix}$ b) $A = \begin{bmatrix} -\frac{4}{3} & \frac{5}{3} \\ -\frac{5}{3} & \frac{4}{3} \end{bmatrix}$ c) $A = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix}$ d) $A = \begin{bmatrix} \frac{4}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{4}{3} \end{bmatrix}$

e) $A = \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ \frac{4}{3} & -\frac{5}{3} \end{bmatrix}$ f) $A = \begin{bmatrix} \frac{4}{3} & -\frac{5}{3} \\ \frac{5}{3} & -\frac{4}{3} \end{bmatrix}$ g) none of the above

$$\det(A^{10} + A) = \boxed{}$$

6. Identify all possible eigenvalues of an $n \times n$ matrix A if A which satisfies the following matrix equation:

$$A - 2I = -A^2.$$

Must A be invertible? Record your answer in the box below and provide justification for your answer.

a) $\lambda = 0, 1$ b) $\lambda = 0, 2$ c) $\lambda = 0, 1, -2$ d) $\lambda = 1, -2$ e) $\lambda = 0, 1, -3$ f) $\lambda = 1, -3$ g) none of the above

Is A invertible? $\boxed{}$

7. Solve the differential equation

$$9x^2y'' + 2y = 0$$

on the interval $(0, \infty)$ subject to the initial conditions $y(1) = 1$ and $y'(1) = \frac{4}{3}$.

a) $y = 2x^{\frac{2}{3}} - 3x^{\frac{1}{3}}$ b) $y = 3x^{\frac{2}{3}} - 2x^{\frac{1}{3}}$ c) $y = 3x^{\frac{3}{2}} - 3x^3$ d) $y = 3x^{\frac{3}{2}} - 2x^3$
e) $y = 2x^2 - 3x$ f) $y = 3x^2 - 2x$ g) none of the above

8. Let $\vec{\omega} := \langle 1, 2, 3\rangle$, and let $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$. Now consider the differential equation

$$\frac{d}{dt}\vec{r} = \vec{\omega} \times \vec{r}.$$

Select the answer which correctly expresses this system of equations in matrix notation when

$$\vec{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}.$$

Do not solve the system.

a) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix} \vec{X}$

b) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix} \vec{X}$

c) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \vec{X}$

d) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} \vec{X}$

e) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix} \vec{X}$

f) $\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix} \vec{X}$

g) none of the above

9. Select the answer below which corresponds to the first few terms in a power series solution of the differential equation

$$x^2y'' + (x^2 - x)y' + y = 0.$$

Will there be a second, linearly independent series solution for this equation? Explain your answer.

a) $y = x^{\frac{1}{2}} + \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{6}x^{\frac{5}{2}} + \frac{1}{12}x^{\frac{7}{2}} + \cdots$
b) $y = -x^{\frac{1}{2}} + \frac{1}{6}x^{\frac{3}{2}} - \frac{1}{12}x^{\frac{5}{2}} + \frac{1}{20}x^{\frac{7}{2}} + \cdots$
c) $y = x^{\frac{1}{2}} + x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{5}{2}} - \frac{1}{9}x^{\frac{7}{2}} + \cdots$

d) $y = x - x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \cdots$
e) $y = x - x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^4 + \cdots$
f) $y = x + x^2 + \frac{1}{2}x^3 + \frac{1}{9}x^4 + \cdots$

g) none of the above

10. Let y be a function satisfying $y(0) = y'(0) = y''(0) = 0$ which is a solution of the ODE

$$y''' - 4y'' + 4y' = 4.$$

Compute $y(1)$.

a) $y(1) = -5$ b) $y(1) = 4$ c) $y(1) = -3$ d) $y(1) = 2$ e) $y(1) = -1$ f) $y(1) = 0$ g) none of the above

11. Solve the following system of differential equations subject to the initial conditions $y_1(0) = 1$ and $y_2(0) = 3$. **Clearly state your solution.** What is $y_1(1)$?

$$\begin{aligned} \frac{dy_1}{dx} &= 3y_1 - y_2 \\ \frac{dy_2}{dx} &= \ \ y_1 + y_2 \end{aligned}$$

a) $y_1(1) = 2e$
b) $y_1(1) = 2e - 1$
c) $y_1(1) = 3$
d) $y_1(1) = 5e^2$
e) $y_1(1) = 7e$
f) $y_1(1) = -e^2$
g) none of the above

12. Find a solution to the initial value problem $y'' - 2xy' - 4y = 0$ subject to the initial conditions $y(0) = 0$ and $y'(0) = 1$ which takes the form of a power series centered at the origin. What is the coefficient in front of x^5 in the series?

a) -1 b) 0 c) $\frac{1}{2}$ d) 1 e) 2 f) 6 g) none of the above

13. Circle "T" for true or "F" for false in the space provided to the left of the following statements. You **DO NOT** need to justify your answer for full credit.

(T F) Every 2×2 diagonalizable matrix with repeated eigenvalue is a diagonal matrix.

(T F) There is a vector field $\vec{F}$ such that $\nabla \times \vec{F} = \langle x, y, z \rangle$.

(T F) If $\det(A) = 0$, then the system $A\vec{X} = 0$ has infinitely many solutions.

(T F) If y_1 and y_2 are solutions to a non-homogeneous linear differential equation, then $y_1 + y_2$ is also a solution.

(T F) If A and B are square matrixes such that $AB^2 = I$, then B is invertible.

Scratch Work Page—Will Not Be Graded If Detached