

### MATH 115 – Sample Final Exam 4

- Ten equally-qualified applicants, 6 men and 4 women, apply for 3 lab technician positions. Unable to justify choosing any of the applicants over the others, the personnel director decides to select 3 at random. What is the probability that one man and two women will be chosen?  
(a) 0      (b)  $\frac{1}{6}$       (c)  $\frac{1}{30}$       (d)  $\frac{1}{2}$       (e)  $\frac{3}{10}$       (f)  $\frac{1}{10}$       (g)  $\frac{3}{7}$       (h)  $\frac{2}{9}$
- A random variable  $X$  on the interval  $[1, \infty)$  has probability density function  $f(x) = \frac{3}{x^4}$ . The variance of this random variable is  
(a)  $\frac{3}{2}$       (b) 3      (c)  $\frac{5}{4}$       (d)  $\frac{21}{4}$       (e)  $\frac{3}{4}$       (f) 5      (g)  $\frac{9}{4}$       (h)  $\frac{1}{2}$
- Consider the following experiment: A fair coin is flipped until the first heads appears, and the number of flips is recorded. If this experiment is repeated three times, what is the probability that the result (number of flips) is the same all three times?  
(a)  $\frac{1}{8}$       (b)  $\frac{1}{4}$       (c)  $\frac{3}{8}$       (d)  $\frac{2}{9}$       (e)  $\frac{1}{3}$       (f)  $\frac{1}{7}$       (g)  $\frac{2}{7}$       (h)  $\frac{1}{2}$
- A box contains 3 red balls, 4 white balls, and 3 blue balls. Balls are drawn from the box one at a time, at random, without replacement. What is the probability that all three red balls will be drawn before any white ball is obtained?  
(a)  $\frac{1}{35}$       (b)  $\frac{3}{128}$       (c)  $\frac{1}{60}$       (d)  $\frac{5}{64}$       (e)  $\frac{7}{120}$       (f)  $\frac{27}{128}$       (g)  $\frac{21}{60}$       (h)  $\frac{15}{64}$
- On a day when Tom operates the machinery, 70% of its output is high quality. On a day when Sally operates the machinery, 90% of its output is high quality. Tom operates the machinery 3 days out of 5. Three pieces of a random day's output were selected at random and 2 of them were found to be of high quality. What is the probability that Tom operated the machinery that day?  
(a)  $\frac{3}{5}$       (b)  $\frac{18}{32}$       (c)  $\frac{27}{49}$       (d)  $\frac{36}{125}$       (e)  $\frac{49}{67}$       (f)  $\frac{61}{84}$       (g)  $\frac{105}{188}$       (h)  $\frac{125}{201}$
- Suppose  $X$  and  $Y$  are independent, exponentially distributed random variables with mean  $\mu$ . What is the expected value of the minimum of  $X$  and  $Y$ ? (For example, if the average waiting time for an elevator is  $\mu$  minutes, what is your expected waiting time if there are two such elevators?)  
(a)  $\mu$       (b)  $2\mu$       (c)  $\sqrt{2}\mu$       (d)  $(\ln 2)\mu$       (e)  $e^2\mu$       (f)  $e^{-2}\mu$       (g)  $\frac{\mu}{2}$       (h)  $\frac{\mu}{\sqrt{2}}$

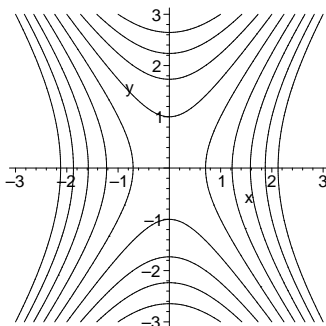
7. At a local carnival, Osa pays \$1 to play a game in which she chooses a card at random from a standard deck of 52 cards. If she chooses a heart, then she receives \$2 (that is, \$1 plus her initial bet of \$1). If she chooses the Queen of Spades she receives \$13. Which of the following is closest to Osa's expected net profit from playing the game?

- (a) -\$0.29                      (b) -\$0.25                      (c) -\$0.10                      (d) \$0.00  
 (e) \$0.10                        (f) \$0.29                        (g) \$0.48                        (h) \$0.75

8. The scores on a Math 115 final exam are normally distributed with a mean of 60 and a standard deviation of 20. A score between 50 and 80 is given a letter grade of B. If there are 200 students in the class, then the expected number of B's falls within which of the following ranges?

- (a) 30–50                        (b) 50–70                        (c) 70–90                        (d) 90–110  
 (e) 110–130                      (f) 130–150                      (g) 150–170                      (h) 170–200

9. The following is a contour plot of  $z = f(x, y)$ .



What is  $f(x, y)$ ?

- (a)  $x^2 + y^2$                       (b)  $2x^2 + y^2$                       (c)  $x^2 + 2y^2$                       (d)  $x^2 - y^2$   
 (e)  $x^2 - 2y^2$                       (f)  $2x^2 - y^2$                       (g)  $xy$                               (h)  $x + y$

10. Let  $f(x, y) = x^2 + 4xy + 2y^4$ . What can you tell from the first and second derivative tests about the critical points  $P = (0, 0)$  and  $Q = (2, -1)$  of  $f$ ?

- (a) saddle at  $P$ , max at  $Q$                       (b) inconclusive at  $P$ , max at  $Q$   
 (c) max at  $P$ , max at  $Q$                               (d) min at  $P$ , max at  $Q$   
 (e) saddle at  $P$ , min at  $Q$                       (f) inconclusive at  $P$ , min at  $Q$   
 (g) max at  $P$ , min at  $Q$                               (h) min at  $P$ , min at  $Q$

11. Let  $f(x, y) = \sqrt{x^2 + y^2}$ . Note that  $f(3, 4) = 5$ . Approximating  $f(3.2, 3.9)$  using the differential of  $f$  at  $(3, 4)$ , one gets:

- (a) 5.04    (b) 5.02    (c) 4.98    (d) 5.10    (e) 4.96    (f) 4.40    (g) 5.00    (h) 5.80

12. Find the maximum of the function  $f(x, y) = 2x + y$  subject to the constraint  $2x^2 + y^2 = 3$ .

- (a) 3      (b)  $\sqrt{3}$       (c)  $2\sqrt{3}$       (d) 1      (e) 2      (f)  $\sqrt{3}/2$       (g) 6      (h) 9

13. Let  $T$  be the triangle with vertices  $(0,0)$ ,  $(2,0)$  and  $(1,2)$ . Compute the integral

$$\iint_T 3y^2 dA$$

- (a) 0      (b) 1      (c) 2      (d) 3      (e) 4      (f) 6      (g) 12      (h) 15

14. Evaluate  $\int_0^{\sqrt{\ln 5}} \int_y^{\sqrt{\ln 5}} e^{x^2} dx dy$

- (a) 1                              (b)  $e$                               (c) 2                              (d)  $\ln 5$   
(e)  $2 \ln 5$                               (f) 5                              (g)  $e^4$                               (h)  $e^2$

15. For what value of  $k$  does this system of equations have at least one solution?

$$\begin{aligned}x - 2y + z &= 2 \\2x + y - z &= 4 \\4x - 3y + z &= k\end{aligned}$$

- (a) 0      (b) 1      (c)  $-1$       (d) 3      (e) 4      (f) 6      (g) 8      (h) no value

16. Two companies, I and II, compete for the entire market for a certain product. During any given year, company I keeps 70% of its customers and loses 30% to company II, whereas company II keeps 80% of its customers and loses 20% of its customers to company I. Initially, company I has 65% of the market and company II has 35%. What percentage of the market will company II have in the long run?

- (a) 70%      (b) 65%      (c) 45%      (d) 50%      (e) 80%      (f) 55%      (g) 35%      (h) 60%

17. The maximum value of  $P = 2x + 3y$  subject to  $x + y \leq 12$ ,  $x + 3y \leq 18$ ,  $x \leq 10$  and  $x, y \geq 0$  is:

- (a) 0      (b) 18      (c) 20      (d) 24      (e) 26      (f) 27      (g) 28      (h) 36