

MATH 115 – Sample Final Exam 3

1. There are 5 College students, 4 Wharton students and 3 Engineering students who are eligible to serve on a certain committee. If the committee is to consist of 2 College students, 2 Wharton students, and 2 Engineering students, how many different committees are possible?
(a) 60 (b) 120 (c) 180 (d) 1080 (e) 1440
2. Choose k so that the function $f(x) = kx^4$ for $0 \leq x \leq 1$ is a probability density function on the interval $[0,1]$, and then compute the expected value of the distribution so obtained.
(a) $1/2$ (b) $2/3$ (c) $3/4$ (d) $4/5$ (e) $5/6$
3. Of the 120 students in a class, 30 speak Chinese, 50 speak Spanish, 75 speak French, 12 speak Spanish and Chinese, 30 speak Spanish and French, and 15 speak Chinese and French. Seven students speak all three languages. What is the probability that a randomly chosen student speaks none of these languages?
(a) 0 (b) $1/10$ (c) $1/8$ (d) $1/6$ (e) $1/4$
4. There are three coins in a box. When tossed, one of the coins comes up heads only 30% of the time, one of the coins is fair, and the third comes up heads 80% of the time. A coin is selected at random from the box and tossed three times. If two heads and a tails comes up (in *that order*), what is the probability that the coin was the fair coin?
(a) $81/195$ (b) $125/316$ (c) $216/541$ (d) $385/902$ (e) $7/11$
5. Two points are selected at random (independently and uniformly) along a 1-meter-long stick, and the stick is broken at the two selected points. What is the probability that the three pieces so obtained can be assembled into a triangle (i.e., that the length of the longest of the three pieces is less than the sum of the lengths of the other two, i.e., that none of the pieces is more than 0.5 meter in length)?
(a) $1/4$ (b) $1/2$ (c) $2/3$ (d) $3/4$ (e) 1
6. The length of time it takes to wait for the bus is an exponentially distributed random variable (i.e., its probability density function is $f(x) = ke^{-kx}$. If the probability that you have to wait more than 10 minutes is $1/e$, then what is the expected waiting time?
(a) e^{-10} min (b) $e^{-1/10}$ min (c) $e^{1/10}$ min (d) $1/10$ min (e) 10 min

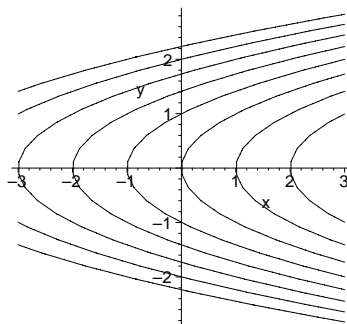
7. Suppose X is a random variable, uniformly distributed on the interval $[0,3]$. What is the expected value of 10^X ?

- (a) 500 (b) $10\sqrt{10}$ (c) $\frac{333}{\ln 10}$ (d) $500 \ln 10$ (e) $\frac{1000}{\ln 10}$

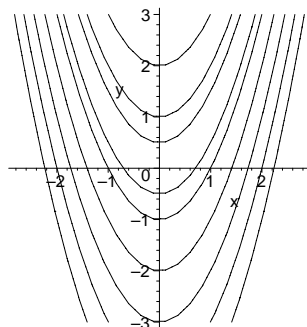
8. Suppose that the lifetime of a certain kind of memory chip is normally distributed with a mean of 400 hours and a standard deviation of 25 hours. If you buy three of them and operate them independently, what is true about the probability p that at least one of the chips lasts at least 450 hours?

- (a) $0 < p < 0.1$ (b) $0.1 < p < 0.25$ (c) $0.25 < p < 0.5$
 (d) $0.5 < p < 0.75$ (e) $0.75 < p < 0.9$

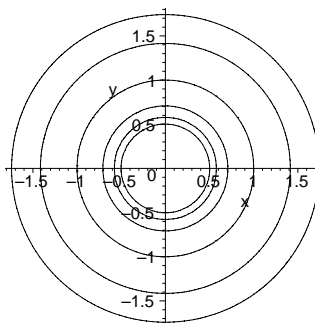
9. Each of the graphs below is the contour plot showing the level curves of one of the listed functions. Match the functions with their contour plots.



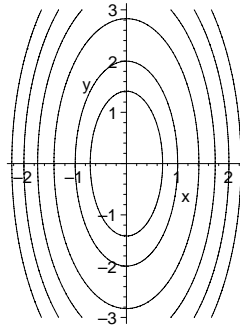
(i)



(ii)



(iii)



(iv)

(a) $f(x, y) = x^2 + \frac{1}{4}y^2$

(b) $f(x, y) = \frac{1}{x^2 + y^2}$

(c) $f(x, y) = x^2 - y$

(d) $f(x, y) = x - y^2 + 3$

10. The function $f(x, y) = x^2 + 3xy + y^2 - 5x - 5y$ has one critical point. Find it and determine its type.

(a) max at (1,1)

(b) min at (1,1)

(c) saddle at (1,1)

(d) max at (2,2)

(e) saddle at (2,2)

11. What is the maximum value of the function $f(x, y) = 4xy$ subject to the constraint $x^2 + 4y^2 = 9$?

(a) 9

(b) 4

(c) 3

(d) 2

(e) 1

12. Find the straight line that best fits through the points (1,2), (2,11) and (3,8).

(a) $y = 2x$

(b) $y = x + 2$

(c) $y = 2x + 4$

(d) $y = 3x$

(e) $y = 3x + 1$

13. Evaluate the double integral of $f(x, y) = y/x$ over the region that lies between the line $y = 3x$ and the parabola $y = x^2 - 3x + 8$.

(a) $15 - 10 \ln 2$

(b) $36 - 25 \ln 2$

(c) $26 - 32 \ln 2$

(d) $52 - 15 \ln 2$

(e) $22 - 40 \ln 2$

14. Find the value of k so that the following matrix has no inverse:

$$\begin{bmatrix} -2 & 5 & 2 \\ 1 & -3 & -1 \\ -1 & 2 & k \end{bmatrix}$$

(a) $k = 0$

(b) $k = 1$

(c) $k = -1$

(d) $k = 2$

(e) the matrix is always invertible

15. How many solutions does the following system of equations have?

$$\begin{aligned}x + 2y + 3z &= 4 \\5x + 6y + 7z &= 8 \\x - z &= -2\end{aligned}$$

- (a) none (b) one (c) two (d) three (e) infinitely many
16. A regional planning board knows that although the population of their region remains stable, each year 30% of the people in the eastern portion move to the west, and 40% of the people in the western section move to the east. What is the fraction of the people in the region who, in the long run, live in the west?
- (a) 3/10 (b) 4/10 (c) 3/7 (d) 4/7 (e) 7/10
17. The maximum value of $P = x + y$ subject to $2x + 3y \leq 50$, $3x + 2y \leq 50$, and $x, y \geq 0$ is
- (a) 12 (b) 50/3 (c) 20 (d) 25 (e) 50
18. Infotron, Inc. makes electronic hockey and soccer games. Each hockey game requires 2 labor-hours of assembly and 2 labor-hours of testing. Each soccer game requires 3 labor-hours of assembly and 1 labor-hour of testing. Each day there are 42 labor-hours available for assembly and 26 labor-hours available for testing. Find the maximum total number of games (hockey + soccer) that Infotron can produce each day.
- (a) 13 (b) 14 (c) 15 (d) 16 (e) 17