MATH 115 – Sample Final Exam 1

1. A school has 7 men and 5 women on its faculty. What is the probability that women will outnumber men on a randomly selected five-member committee?

   (a) 240/793 (b) 41/132 (c) 22/61 (d) 82/133 (e) 1/2

2. A continuous random variable X on the interval $[-1, 1]$ has the density function $1 - |x|$. What is the expected value of X^2?

   (a) 0 (b) 1/6 (c) 1/4 (d) 1/2 (e) 2/3

3. A casino game consists of a single toss of a fair die and pays off as follows: if the die comes up with an odd number, the player is paid that number of dollars (i.e., \$1 for rolling a 1, \$3 for rolling a 3, and \$5 for rolling a 5), and if an even number comes up the player is paid nothing. What fee should the casino charge to play the game to make it exactly fair?

   (a) \$1.50 (b) \$2.00 (c) \$2.50 (d) \$3.00 (e) \$3.50

4. A supermarket buys its eggs from three different chicken ranches. They buy 1/3 of their eggs from Eggs'R Us, 1/2 of their eggs from The Yolk Ranch, and 1/6 of their eggs from Cheap Eggs. The supermarket determines that 1% of the eggs from Eggs'R Us are cracked, 2% of the eggs from the Yolk Ranch are cracked, and 5% of the eggs from Cheap Eggs are cracked. What is the probability that an egg chosen at random is from Cheap Eggs, given that the egg is cracked?

   (a) 13/600 (b) 1/20 (c) 13/100 (d) 5/13 (e) 7/11

5. The length of time between calls received by the switchboard in a large legal firm in Phoenix is an exponential random variable. The average length of time between calls is 20 seconds. If a call has just been received, what is the probability that no calls will be received in the next 30 seconds?

   (a) 3/2 (b) 0 (c) $1 - e^{3/2}$ (d) $e^{-2/3}$ (e) $e^{-3/2}$

6. In a certain book, there is 1 misprint per 2 pages, on the average. What is the probability that there are 2 or more misprints on a given, randomly chosen page? (This is a Poisson process.)

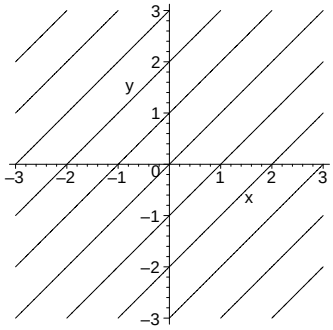
   (a) $1 - \frac{3}{2}e^{-1/2}$ (b) $1 - \frac{1}{2}e^{-1/2}$ (c) $1 - \frac{3}{2}e^{-2}$ (d) $\frac{1}{2}e^{-1/2}$ (e) $\frac{1}{2} - \frac{3}{2}e^{-1/2}$

7. In a certain state, the tax returns of everyone whose income is in the top 15% of incomes are audited. Assume that income in this state is normally distrubuted with mean $\mu = \$54,000$ and standard deviation $\sigma = \$15,000$. The minimum income in the audited group is closest to which of the following?

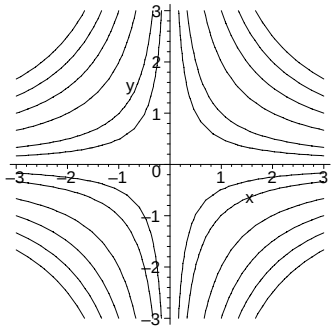
   (a) \$38,400 (b) \$39,200 (c) \$69,600 (d) \$72,200 (e) \$84,800

8. Each of the graphs below is the contour plot showing the level curves of one of the listed functions. Match the functions with their contour plots.

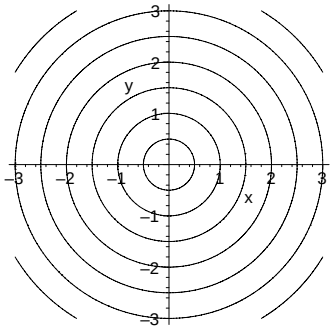
(i)



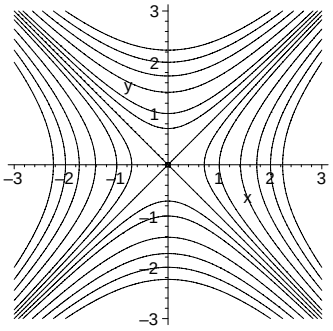
(ii)



(iii)



(iv)



(a) $f(x, y) = x^2 + y^2$ (b) $f(x, y) = x^2 - y^2$ (c) $f(x, y) = xy$ (d) $f(x, y) = x - y$

9. The tangent plane to the graph of $z = 3x^2y - e^{xy} + 4y$ at the point (0,1,3) is

   (a) $z = 4x + y + 2$ (b) $z = -x + 2y + 1$ (c) $z = -x + 4y - 1$
   (d) $z = 2x - 4y + 7$ (e) $z = 2x + 4y - 1$

10. The graph of the function $f(x, y) = y^3 - x^2 + 6x - 12y + 5$ has exactly one saddle point. The value of $f(x, y)$ at the saddle point is:

    (a) -38 (b) -24 (c) -2 (d) 0 (e) 24

11. Find the point(s) closest to (1,0) on the ellipse $x^2/3 + y^2 = 1$.

    (a) (1,0) (b) $(0, \pm 1)$ (c) $(\sqrt{3}, 0)$ (d) $(\pm 3/2, 1/4)$ (e) $(3/2, \pm 1/2)$

12. Evaluate the double integral of $f(x, y) = 2xy$ over the region in the half-plane $x > 0$ that lies between the parabolas $y = x^2$ and $y = 2 - x^2$.

    (a) 3 (b) 1/2 (c) 1 (d) -1 (e) 0

13. The value of $\displaystyle\int_0^1 \int_{y^2}^1 ye^{x^2}\, dx\, dy$ is equal to:

    (a) $e - 1$ (b) $\frac{1}{4}(e - 1)$ (c) $\frac{1}{4}e$ (d) $\frac{1}{2}(e - 1)$ (e) $e - \frac{1}{2}$

14. Which of the following systems of linear equations have infinitely many solutions?

$$\text{(I)} \begin{cases} x - 2y + 5z = 7 \\ y - 2z = 4 \end{cases} \quad \text{(II)} \begin{cases} x - y + 4z - 3w = 2 \\ z + 2w = 8 \\ w = 3 \end{cases}$$

$$\text{(III)} \begin{cases} x + y - z = 1 \\ x - 2z = 1 \\ 3y - 6z = 4 \end{cases} \quad \text{(IV)} \begin{cases} x + 2y - z = 2 \\ x - z = 3 \\ 2x - 2z = 6 \end{cases}$$

    (a) I, II and IV only (b) I, II, III and IV (c) I,II and III only
    (d) I and II only (e) III and IV only

15. Suppose M is a 2×2 matrix. If $M^2 = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$ and $M^5 = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$, find M.

    (a) $\begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

16. A manufacturer produces two items, A and B. A maximum of 2,000 units (total between the two items) can be produced in a day. It costs \$3 to produce one of item A and \$5 to produce one of item B. Daily production costs must not total more than \$7,500. If the manufacturer makes a profit of \$1.75 per unit of A and \$2.50 per unit of B, how many units of A should he produce to maximize the daily profit?

    (a) 750 (b) 1050 (c) 1250 (d) 1425 (e) 2500

17. The maximum value of $P = 2x + 3y + 4z$ subject to $x + z \leq 4$, $y + z \leq 3$ and $x, y, z \geq 0$ is

    (a) 7/12 (b) 84 (c) 12 (d) 12/7 (e) 17

18. Using data collected for a particular region over many years, an insurance company has ascertained that 20% of the drivers involved in an automobile accident one year are also involved in an accident the following year, while only 10% of the drivers not involved in an accident one year are involved in an accident the following year. Use these percentages as approximate empirical probabilities to find the probability that (in the long run) a driver chosen at random will be involved in an accident during any given year.

    (a) 1/20 (b) 1/12 (c) 1/10 (d) 1/9 (e) 1/8