Math 115 Final Exam Fall 2006

1. Consider the surface $z = f(x, y) = 2x^2 + y^2$. Find the tangent plane to the surface at the point $(x, y, z) = (1, 1, 3)$ and find where this plane intersects the z-axis. Plane intersects the z-axis at $z =$

$$A.\ 3 \quad B.\ 2 \quad C.\ 0 \quad D.\ -3 \quad E.\ -4 \quad F.\ -6 \quad G.\ -8 \quad H.\ 10.$$

2. Find the shortest distance from the origin to the plane $2x - 2y + z = 9$.

$$A.\ 1 \quad B.\ \sqrt{2} \quad C.\ \sqrt{3} \quad D.\ 2 \quad E.\ \sqrt{5} \quad F.\ 3 \quad G.\ \sqrt{8} \quad H.\ 4$$

3. Consider the function $f(x, y) = x^2 - 4x + y^3 - 3y$. Find its critical points and determine their type.

A. Local min at (2, 1), local min at (2, -1).
B. Local min at (2, 1), saddle at (2, -1).
C. Local min at (2, 1), local max at (2, -1).
D. Saddle at (2, 1), local min at (2,-1).
E. Saddle at (2,1), saddle at (2,-1).
F. Saddle at (2,1), local max at (2, -1).
G. Local max at (2, 1), local min at (2, -1).
H. Local max at (2, 1), Saddle at (2, -1).

4. If $z = e^x \sin(y)$, where $x = st^2$ and $y = s^2 t$, then $\frac{\partial z}{\partial s}$ when $s = 1, t = 1$ is:

$$A.\ 0 \quad B.\ \sin(1) \quad C.\ \cos(1) \quad D.\ e\sin(1) \quad E.\ e\cos(1)$$
$$F.\ 2e\cos(1) \quad G.\ e\sin(1) + 2e\cos(1) \quad H.\ e\sin(1) - 2e\cos(1).$$

5. Evaluate

$$\int_0^2 \int_y^2 e^{-\frac{x^2}{2}}\, dx dy$$

$$A.\ \frac{e^2 - 1}{8} \quad B.\ 2\ln(2) - 1 \quad C.\ \frac{1 - e}{2} \quad D.\ 1 - e^{-2}$$
$$E.\ \frac{e^2 - 1}{4} \quad F.\ 0 \quad G.\ e\sqrt{2} - 1. \quad H.\ \ln(2)$$

6. In a lottery, the players select 6 numbers between 1 and 49. What is the probability that the winning combination consists of all odd numbers ?

$$A.\ \frac{\binom{25}{6}}{P_{25,6}} \quad B.\ \frac{\binom{24}{6}}{P_{25,6}} \quad C.\ 25/49 \quad D.\ 1/2 \quad E.\ \frac{\binom{24}{6}}{\binom{49}{6}} \quad F.\ \frac{\binom{25}{6}}{\binom{49}{6}} \quad G.\ \frac{\binom{49}{2}}{\binom{49}{6}} \quad H.\ \frac{P_{49,2}}{\binom{49}{6}}$$

7. A jar contains 6 red balls and 4 green balls. If three balls are selected at random (without replacement) what is probability that there are more red balls than green balls?

$A.$ 3/10 $B.$ 1/2 $C.$ 3/5 $D.$6/11 $E.$ 2/3 $F.$ 7/11 $G.$ 83/120 $H.$ 7/13

8. 6% of Type A light bulbs are defective, 4% of Type B light bulbs are defective, and 2% of Type C light bulbs are defective. A light bulb is selected at random from a batch of light bulbs containing 50 Type A bulbs, 30 Type B bulbs, and 20 Type C bulbs. The selected bulb is found to be defective. What is the probability that the selected bulb was of Type A?

$A.$ 3/4 $B.$ 10/23 $C.$ 1/3 $D.$ 2/3 $E.$ 15/23 $F.$ 2/23 $G.$ 1/2 $H.$ 7/10

9. X is a continuous random variable on the interval [0,1] whose density function is of the form $k - kx$ for some constant k. What is Var(X)?

$A.$ 1/6 $B.$ 1/2 $C.$ 1/18 $D.$ 2/3 $E.$ 0 $F.$ 1/3 $G.$ 1/9 $H.$ 13/180.

10. The waiting time for an elevator is an exponentially distributed random variable with mean 3 minutes, i.e., it has probability density function $f(x) = \frac{1}{3}e^{-\frac{x}{3}}$ for $x \geq 0$. When your guest arrives at your floor you ask if he had to wait longer than 6 minutes for the elevator. He says no. What is the probability that he had to wait at least 3 minutes?

$A.$ 1/2 $B.$ $(e-1)/(e-2)$ $C.$ e^{-1}/e^{-2} $D.$ e^{-2}/e^{-1} $E.$ e^{-2}
$F.$ $e^{-1} - e^{-2}$ $G.$ $(e^{-1} - e^{-2})/(e^{-2} - 1)$ $H.$ $(e^{-1} - e^{-2})/(1 - e^{-2})$

11. A fair coin is tossed 8 times. What is the probability that the number of heads in the first four tosses equals the number of heads in the last fours tosses? (e.g. HTTHHHTT they are equal, HHTHTTHH they are not equal. Note that $2^4 = 16,\ 2^5 = 32,\ 2^6 = 64,\ 2^7 = 128,\ 2^8 = 256$)

$A.\ \frac{92}{256}$ $B.\ \frac{70}{256}$ $C.\ \frac{54}{256}$ $D.\ \frac{50}{256}$ $E.\ \frac{42}{256}$ $F.\ \frac{36}{256}$ $G.\ \frac{32}{256}$ $H.\ \frac{28}{256}$

12. The number of clicks of a Geiger counter is a Poisson process with a mean of 2 clicks per minute. What is the probability there are three or more clicks in 2 minutes.

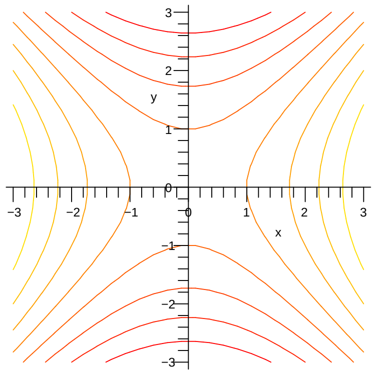
$A.\frac{2}{e}$ $B.1-2/e$ $C.1-5e^{-2}$ $D.1-3e^{-3}$ $E.\ 1/e$ $F.1-13e^{-4}$ $G.12e^{-4}$ $H.1-21e^{-4}$

13. A gambler play a certain game. For each play, the gambler will get \$10 if he win and lose \$10 if he lose. Suppose that the probability to win is p $(0 \le p \le 1)$ and he begins playing with a given fortune \$100, what is the expected value after n independent plays of game?

$$A.\ 100 + 10np(1-p) \quad B.\ 100 + 10n(1-2p) \quad C.\ 100 + 10n(2p-1)$$

$$D.\ 100 + 10np(2p-1) \quad E.\ 100 + 10np(1-3p) \quad F.\ 100 + 10n\frac{p}{1-p}$$

14. The following is a contour plot of $z = f(x, y)$.



What is $f(x, y)$?

$$A.\ x^2 + y^2 \quad B.\ 2x^2 + y^2 \quad C.\ x^2 - 10y^2 \quad D.\ x^2 - y^2$$

$$E.\ 2xy \quad F.\ x^2 - y \quad G.\ xy \quad H.\ x^2 + y$$

15. Which of the following statements regarding the system of equations

$$\begin{cases} x - y + z = 1 \\ x + 2y + 3z = 4 \\ 2x + 4y + 6z = k \end{cases}$$

is true ?

(1) The system has a unique solution for any value of k.
(2) The system has a unique solution only when $k = 8$.
(3) The system has a unique solution only when $k = 0$.
(4) The system only has infinitely many solutions when $k = 8$.
(5) The system infinitely many solutions for every value of k.
(6) The system has a unique solution for $k = 0$ and infinitely many when $k = 8$.
(7) The system never has a solution.
(8) The system has infinitely many solutions when $k > 0$, a unique solution when $k = 0$, and no solutions for $k < 0$.

16. Given that

$$A = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix} \text{ and } A \cdot B = \begin{pmatrix} 1 & 2 \\ 2 & -4 \end{pmatrix} \text{ find } B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and compute the product $P = ad$. Then $P =$

$$A.\ 2 \quad B.\ -2 \quad C.\ 1 \quad D.\ -1 \quad E.\ 0 \quad F.\ 3 \quad G.\ -3 \quad H.\ 4$$

17. A bank teller has a total of 70 bills in five-, ten-, and twenty-dollar denominations. The number of fives is three times the number of tens, while the total value of the money is \$960. Find the number (x_5, x_{10}, x_{25}) of each type of bill.

A. (20, 15, 18) B. (20, 8, 18) C. (24, 8, 17) D. (20, 50, 12)
E. (25, 15, 7) F. (10, 15, 18) G. (12, 8, 33) H. (24, 8, 38)

18. Three rods A, B and C are to be welded end to end to make 5 meter rod. The lengths of each of the rods is a normal distributed random variable with means μ and standard deviations σ given in the table below. What is the probability that the assembled rods will be within 1 millimeters of 5 meters? (i.e. $\Pr(|X_A + X_B + X_C - 5m| < 1cm)$ (1 centimeter = 10^{-2}meters)

$$A\ \mu = 1 \text{ meters} \quad \sigma = 1 \text{ centimeters}$$
$$B\ \mu = 2 \text{ meters} \quad \sigma = 2 \text{ centimeters}$$
$$C\ \mu = 2 \text{ meters} \quad \sigma = 2 \text{ centimeters}$$

In the following answers, $\phi(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt$.

A. $\phi(\frac{1}{3})$ B. $\phi(\frac{1}{9})$ C. $2\phi(0.01)$ D. $2\phi(\frac{1}{3})$
E. $\phi(\frac{1}{5})$ F. $2\phi(0.02)$ G. $1 - \phi(\frac{1}{3})$ H. $1 - \phi(0.01)$.

Answer Key:
1. D 2. F 3. B 4. G 5. D 6. F 7. E 8. E 9. C 10. H 11. B 12. F 13. C 14. D 15. D 16. A 17. H 18. D