Math 115 (Powers) Final Exam Tuesday December 17, 2013

1. If $f(x,y,z) = xyz$ and x, y and z are functions of time and at time t = 0 $(x,y,z) = (1,2,3)$ and $(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}) = (1,1,1)$ what is $\frac{dh}{dt}$ at time t = 0 where $h(t) = f(x(t),y(t),z(t))$. Ans $\frac{dh}{dt}$ =

A. 0 B. 3 C. 5 D. 8 E. 9 F. 11## G. 13 H. none of these

2. Find where the plane tanget to the surface $2x^2 + 2y^2 + 3z^2 = 16$ at the point $(x,y,z) = (1,-1,2)$ intersects the z-axis. Ans z =

A. -1 B. 0 C. 1 D. 5/3 E. 2 F. 8/3## G. 10/3 H. none of these

3. Suppose $f(x,y) = 2x^3 - 3x^2 + y^2 - 4y$. Find the critical points and determine their type.

A. {rel min at x=0,y=2; rel min at x=1,y=2} B. {rel min at x=0,y=2; saddle at x=1,y=2}

C. {rel min at x=0,y=2; rel max at x=1,y=2} D. ## {saddle at x=0,y=2; rel min at x=1,y=2} E. {saddle at x=0,y=2; saddle at x=1,y=2}

F. {saddle at x=0,y=2; rel max at x=1,y=2} G. {rel max at x=0,y=2; rel min at x=1,y=2} H. {rel max at x=0,y=2; saddle at x=1,y=2}

4. Find the sum of the maximum value and the minimum value of the function $f(x,y) = 2x^2 + 2x + 2y^2 - 2y$ in the disk $x^2 + y^2 \leq 2$.

A. -3 B. -1 C. 1 D. 3 E. 5 F. 7## G. 9 H. none of these

5. Compute the integral $\int_{-1}^{0}\int_{\sqrt{y+1}}^{1} e^{-x^3}\,dx\,dy$

A. 1/e B. e-1 C. $\frac{1-e^{-1}}{3}$## D. ln(3) E. 3e F. ½ln(3) G. $1-e^{-3}$ H. none of these

6. There are 4 red socks, 4 white socks and 4 blue socks in a box. Three are drawn out without replacement. What is the probability there is a pair of socks (two the same color)? Probability of a pair =

A. 17/32 B. 39/55## C. 32/35 D. 11/24 E. 8/11 F. 43/72 G. 3/5 H. none of these

7. Four balls numbered 1,2,3,4 are randomly placed it 3 boxes, A, B and C. What is the probability that A and B have the same number of balls. $\text{Prob}(N_A = N_b)$ =

A. 9/81 B. 12/81 C. 19/81## D. 25/81 E. 29/81 F. 32/81 G. 37/81 H. none of these

8. A weighted coin produces heads 2/3 of the time and tails 1/3 of the time. The coin is flipped four times. What is the probability the coin will produce the same number of heads in the first 2 flips as the last 2 flips? Prob =

A. $\frac{11}{27}$## B. $\frac{20}{81}$ C. $\frac{7}{27}$ D. $\frac{22}{81}$ E. $\frac{10}{27}$ F. $\frac{5}{81}$ G. $\frac{40}{81}$ H. none of these

9. There are 3 coins A, B and C. Coin A produces heads with a probability of 1/4 and tails with probability 3/4. Coin B is fair and produces heads and tails with probability 1/2. Coin C produces heads with probability 3/4 and tails with probability 1/4. One of the coins is selected at random and filped 3 times and it produces more heads than tails (i.e. it produces 2 or 3 heads). What is the probability that it was coin A that was selected. Probability of coin A =

A. 5/48## B. 1/8 C. 1/6 D. 3/16 E. 5/32 F. 11/48 G. 1/4 H. none of these

10. Suppose X and Y are independent random variables and each is uniformly distributed on the interval [0,3] (so for example the distribution function f for X is f(x) = 1/3). Calculate the probability that the sum X+Y is within 1 of 3. Prob($|X + Y - 3| < 1$) =

A. 1/4 B. 1/3 C. 2/5 D. 3/7 E. 1/2 F. 5/9## G. 2/3 H. none of these

11. The random variable X is distributed on the interval [0,2] with probability distribution f(x) = x/2 and the random variable Y is distributed on the interval [0,2] with probability distribution g(y) = y/2 and X and Y are independent so (X,Y) is distributed on the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ with probability distribution h(x,y) = xy/4. Compute the variance of X - Y. (Hint X and Y are independent.) Var(X - Y) =

A. 2/9 B. 1/3 C. 4/9## D. 5/9 E. 3/7 F. 10/7 G. 2 H. none of these.

12. The number of clicks per minute of a Geiger counter is a Poisson process. If the expected number of clicks is 2 clicks per minite what is the probability that there are more than 2 clicks in a given minute. Prob ($N \geq 3$) =

A. $\frac{4e^{-2}}{3}$ B. $1-e^{-1}$ C. $1-5e^{-2}$## D. $1-3e^{-2}$ E. $\frac{3-4e^{-2}}{3}$ F. $5e^{-1}$ G. $1-7e^{-2}$ H. none of these

13. There are two independent exponentially distributed random varialbes X and Y. The mean waiting time for X is 1 second and the mean waiting time for Y is 2 seconds. What is the probability that Y occurs less than 1 second after X? Prob(Y < X + 1) =

A. $e^{-½}$ B. $1-½e^{-½}$ C. $\frac{3-2e^{-½}}{3}$ ## D. $\frac{2-e^{-½}}{2}$ E. $\frac{2-3e^{-½}}{2}$ F. $1-e^{-1}$ G. $\frac{2-e^{-1}}{3}$ H. none of these

14. For each of the systems of equations below indicate whether there are no solutions (0), a unique solution (1) or infinitely many solutions (∞)

I

$$\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ x + 2y + 2z &= 2 \end{aligned}$$

II

$$\begin{aligned} x + y + 2z + w &= 1 \\ y + z + 2w &= 1 \\ 2z + w &= 4 \\ 2w &= 3 \end{aligned}$$

III

$$\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ x + 2y + 2z &= 3 \end{aligned}$$

Circle one

(0 1 ∞) ## (0 1 ∞) ## (0 1 ∞) ##

15. Suppose R is a 3 x 3 matrix and $R\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+2y+3z \\ y+2z \\ z \end{bmatrix}$. Then $R^{-1}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} =$

A. $\begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$ B. $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ C. $\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$ D. $\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ E. $\begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$ F. $\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ G. $\begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$ ## H. none of these

16. Given A is a 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 8 & 7 \\ 0 & 1 \end{bmatrix}$. What is S = a + b + c + d.

A. 0 B. 1 C. 2 D. 3 E. 4 ## F. 5 F. 6 H. none of these

17. What is the probability that the equations $x + 2y = 1$ and $Ax + By = 3$ have no solutlion for x and y where A and B are the values of a two fair independent dice numbered (1-6)? Probalitiy of no solution =

A. 0 B. 1/36 C. 1/18 ## D. 1/12 E. 1/9 F. 5/18 G. 1/3 H. none of these

18. Abe, Bob and Carl are playing catch. Abe only throws to Bob. Bob throws to Abe and Carl with equal probability and Carl throws to Abe and Bob with equal probability. In the long run who has the ball the most and what fraction of the time does he have the ball? Cirle answer.

Abe 7/18
Abe 4/9
Abe 1/2
Bob 7/18
Bob ## 4/9
Bob 1/2
Carl 7/18
Carl 1/2
None of these

19. Suppose $X_1, X_2, \cdots, X_{48}$ are 48 independent random variable where each variable is uniformly distributed on the interval [0,1]. (This means that the distribution function f_k for X_k is $f_k(x) = 1$ for $0 \leq x \leq 1$.) Using the table at the end if the exam estimate the probablity that the sun $S = X_1 + X_2 + \cdots + X_{48}$ rounded to the nearest integer is 24. (That is compute $Pr(|S - 24| < 1/2)$. Show what you looked up and how you used it (e.g. $\varphi(1) - \varphi(-1.5)$). Circle the closest answer.

A. 5% B. 10% C. 15% D. 20% ## E. 25% F. 30% G. 35% H. 40%

Table of the Standard Normal Distribution

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}t^2}\, dt \quad = \quad \text{Prob}(0 < Z < z)$$

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

table by Robert T. Powers