

Math 115 (Powers) Final Exam. Tuesday May 10, 2011

Name(print) _____ Penn I.D. _____

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1. Consider the surface $x^4 + 2y^4 + 4z^2 = 7$. Find the equation for the plane tangent to this surface at $(x,y,z) = (1,1,1)$ and determine where the plane intersect the x-axis. The plane intersects the x-axis at $x =$
A. -3 B. -1 C. 0 D. 1 E. 3 F. 5
2. Find the x-coordinate of the point on the plane $3x + 2y - z = 7$ which is closest to the origin. The closest point has coordinates (x,y,z) with $x =$
A. $3/2$ B. $4/3$ C. $6/5$ D. 1 E. $2/3$ F. 0
3. The function $f(x,y) = y^3 - x^2 + 2x - 12y + 5$ has two critical points one of which is a saddle point. If (x_0,y_0) is the coordinate of the saddle point then find the sum $x_0+y_0 =$
A. 5 B. 3 C. $5/2$ D. $3/2$ E. 1 F. 0
4. Evaluate $\int_0^1 \int_{y^2}^1 ye^{x^2} dx dy$ (You will need to interchange the order of integration.)
A. $\frac{e^2 - 1}{8}$ B. $2\ln(2)-1$ C. $\frac{1}{2}(e + 1)$ D. $1+e^{-2}$ E. $\frac{e - 1}{4}$ F. 0
5. Two red balls, two green balls and a yellow ball are placed in an urn. You select three balls at random. Find the probability that no color occurs more than the green in the three selected.
A. $1/10$ B. $3/10$ C. $7/10$ D. $1/5$ E. $2/5$ F. $1/2$
6. Two unfair coins A and B with probability $2/5$ and $4/5$ of heads, respectively, lie on a table - you cannot tell them apart by appearance. You select one at random and flip it five times, resulting in four heads. What is the probability that is is coin A?
A. $1/13$ B. $2/17$ C. $3/19$ D. $4/21$ E. $5/23$ F. $6/25$

7. Three black cards are mixed with twelve red cards at random producing a deck of 15 cards. A card is drawn from the deck, observed and replaced. This experiment is repeated four times. What is the expected number of black cards you observe?
- A. $4/7$ B. $4/5$ C. 1 D. $5/4$ E. $7/4$ F. 2
8. Three red marbles and five blue marbles are placed in a bag. A marble is drawn from the bag: if it is red, it is set aside and another is drawn. If it is blue, the drawing ceases. What is the expected number of draws?
- A. 1 B. $5/4$ C. $3/2$ D. $7/4$ E. 2 F. $5/2$
9. A fair coin is flipped six times. What is the probability that the number of heads on the first four flips is greater than the number of heads on the last two?
- A. $7/16$ B. $3/16$ C. $15/32$ D. $9/16$ E. $21/32$ F. $13/16$
10. Coin A produces head $2/5$ of the time and coin B produces heads $4/5$ of the time. Each coin is flipped 5 times. Let X_A be the number of heads coin A produces and X_B be the number of heads coin B produces. Let $Z = 2X_A - X_B$. Note the expected value of Z is zero $E(Z) = 0$. What is the variance of Z. $\text{Var}(Z) =$
- A. $8/5$ B. 4 C. 5 D. $28/5$ E. 6 F. $20/3$
11. Suppose X is a continuous random variable distributed on the interval $[0,3]$ with probability distribution $f(x) = 2x/9$. Compute the conditional probability that X lies between 2 and 3 given that X lies between 1 and 3. (i.e. Compute $\Pr(2 < X < 3 | 1 < X < 3)$). $\Pr(2 < X < 3 | 1 < X < 3) =$
- A. $1/4$ B. $1/3$ C. $5/8$ D. $3/4$ E. $4/5$ F. $7/8$
12. The number of clicks per minute of a Geiger counter is a Poisson process with an average counting rate of 3 clicks per minute. Given that there are three or less clicks in the first minute what is the expected number of clicks?
- A. $1 - 14e^{-3}$ B. 3 C. $28/9$ D. $7e^{-3}$ E. $35/17$ F. $51/26$

13. The time T is an exponentially distributed random variable with mean waiting time 3 seconds. Given that no click occurs in the first two seconds what is the probability the first click occurs in the first 4 seconds. $\Pr(T < 4 | T > 2) =$

A. $1 - e^{-1}$ B. $e^{-2/3}$ C. $1 - e^{-2/3}$ D. $e^{-4/3}$ E. $1 - e^{-4/3}$ F. $1 - e^{-1/2}$

14. The equation for a parabola is $y = ax^2 + bx + c$. Given the parabola passes through the three points $(x,y) = (-1,9)$, $(1,5)$ and $(2,12)$ find a . (Note that the fact that the parabola passes through three points gives you three equations for a , b , and c). Ans $a =$

A. -3 B. -2 C. -1 D. 1 E. 2 F. 3

15. For what value of k does the system of equations have at least one (one or more) solutions.

$$x - 2y + z = 2$$

$$2x + y - z = 4$$

$$4x - 3y + z = k$$

A. 2 B. 4 C. 6 D. 8 E. 10 F. no solution for any k .

16. Consider the equations $2x + 2y = 4$ $x + Ay = B$ where A and B are independently chosen random integers from 1 to 10. (i.e. there are 100 equally probably possibilities for the pair (A,B)). What is the probability that these equations have no solution?

A. 0% B. 5% C. 9% D. 19% E. 25% F. 100%

17. Find the value of x determined by the 3 equations

$$3x + y + 2z = 1$$

$$2x - y + 2z = 1$$

$$4x + 5y + z = 1$$

Hint. if $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 5 & 1 \end{bmatrix}$ then $A^{-1} = \begin{bmatrix} -11 & 9 & 4 \\ 6 & -5 & -2 \\ 14 & -11 & -5 \end{bmatrix}$

Ans. $x =$

A. -1 B. 0 C. 1 D. 2 E. 3 F. there is no solution

18. A salesman sells only in three cities A, B, and C. He never sells in the same city two successive days, he sells in city B if he sold in city A the day before and if he sells in B or C he is twice as likely to sell in city A as the other city. In the long run, how often does he sell in B?

A. $3/20$ B. $3/10$ C. $9/20$ D. $6/10$ E. $15/20$ F. $9/10$

19. A fair coin is flipped 200 times and produces X heads. Then the coin is flipped another 200 times and produces Y heads. What is the probability that the absolute value of the difference is less than 15. (Use the table at the end of the exam and indicate what you looked up and how you used it.) Circle the closest answer.

A. 4% B. 20% C. 35% D. 55% E. 65% F. 80%

Answers FABEF CBCED CFCFD CDCD

Table of the Standard Normal Distribution

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}t^2} dt = \text{Prob}(0 < Z < z)$$

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

table by Robert T. Powers