Math 115, Spring 2010 A. Scedrov, K. Zhu

Final Exam

May 4, 2010

Name:______________________________
Penn ID #:____________________________

Show all your work. A correct answer without supporting work receives little or no credit!

	Full score	Your score
Problem 1	10	
Problem 2	10	
Problem 3	10	
Problem 4	10	
Problem 5	10	
Problem 6	10	
Problem 7	10	
Problem 8	10	
Problem 9	10	
Problem 10	10	
Problem 11	10	
Problem 12	10	
Problem 13	10	
Problem 14	10	
Problem 15	10	
Problem 16	10	
Problem 17	10	
Problem 18	10	
Problem 19	10	
Total	190	

1. Consider the surface $x^2 - y^2 + 3z^3 = 3$. Find the equation for the plane tangent to this surface at $(x, y, z) = (1, 1, 1)$ and determine where the plane intersects the x-axis. The plane intersects the x-axis at $x =$

(A) $-\frac{2}{9}$ (B) $\frac{2}{9}$ (C) $-\frac{9}{2}$ (D) $\frac{9}{2}$

(E) 1 (F) -1 (G) $-\frac{1}{2}$ (H) $\frac{1}{2}$

2. Find the maximum of the function $f(x, y, z) = x+y+z$ on the surface $2x^2+3y^2+z^2 = 6$. Maximum=

(A) 3 (B) $1 + \sqrt{\frac{2}{3}} + \sqrt{2}$ (C) $\sqrt{11}$ (D) $\frac{1}{\sqrt{11}}$

(E) $\frac{1}{\sqrt{66}}$ (F) $\sqrt{66}$ (G) $\frac{\sqrt{6}}{\sqrt{11}}$ (H) $\frac{\sqrt{11}}{\sqrt{6}}$

3. The function $f(x, y) = x^4 + x^2 + y^3 - 3y$ has two critical points. Find the points and determine their types.
(A) local min at (0,1); local min at (0,-1)
(B) local min at (0,1); local max at (0,-1)
(C) local min at (0,1); saddle at (0,-1)
(D) local max at (0,1); local min at (0,-1)
(E) local max at (0,1); local max at (0,-1)
(F) local max at (0,1); saddle at (0,-1)
(G) saddle at (0,1); local min at (0,-1)
(H) saddle at (0,1); local max at (0,-1)
(I) saddle at (0,1); saddle at (0,-1)

4. $\int_0^1 \int_{-1}^{-\sqrt{y}} e^{x^3}\, dxdy =$

(A) 1 (B) $1-e$ (C) $1-e^{-1}$ (D) $e-1$

(E) $\frac{1}{3}$ (F) $\frac{1}{3}(1-e)$ (G) $\frac{1}{3}(1-e^{-1})$ (H) $\frac{1}{3}(e-1)$

5. There are 2 red balls, 2 green balls and 1 yellow ball in a jar. Three balls are drawn out without replacement. What is the probability that there are more red balls than yellow balls?

(A) 1/10	(B) 1/5	(C) 3/10	(D) 2/5
(E) 1/2	(F) 3/5	(G) 7/10	(H) 4/5

6. There are 2 red balls, 2 green balls and 1 yellow ball in a jar. Three balls are drawn out with replacement. What is the probability that there are more red balls than yellow balls?

(A) 24/125	(B) 32/125	(C) 36/125	(D) 44/125
(E) 48/125	(F) 56/125	(G) 60/125	(H) 68/125

7. Coin A has a probability of 1/3 of producing a heads and coin B is a fair coin. Each coin is flipped three times. Let X denote the number of heads produced by A and let Y denote the number of heads produced by B. Let $Z = X - 2Y$. Then the variance of Z is

(A) 17/12	(B) 1/12	(C) 5/6	(D) 11/3
(E) 113/72	(F) 49/72	(G) 145/144	(H) 17/144

8. There are four coins. One has a probability of 1/3 of producing a heads, two have a probability of 1/4 of producing a heads and one has a probability of 2/3 of producing a heads. One coin is picked at random and flipped twice producing two heads. What is the probability it is one of the coins with a probability of 1/4 of producing a heads?

(A) 9/49	(B) 8/49	(C) 32/49	(D) 81/121
(E) 8/121	(F) 32/121	(G) 9/17	(H) 8/17

9. The continuous random variable X is distributed over the interval $[1, 2]$ with probability density $k(x^2 - x)$. Find the variance of X. $\text{Var}(X) =$

(A) $1/24$	(B) $147/60$	(C) $17/12$	(D) $289/144$
(E) $1/20$	(F) $147/50$	(G) $17/10$	(H) $289/100$

10. A player tosses two fair coins. The player wins \$3 if 2 heads occur and \$1 if 1 heads occurs. For the game to be fair, i.e. for the player's expected gain to be 0, how much should the player lose if no heads occurs? The player's **LOSS** should be:

(A) \$2	(B) \$3	(C) \$6	(D) \$7
(E) \$1	(F) \$8	(G) \$5	(H) \$4

11. Suppose X is a continuous random variable uniformly distributed on the interval [0,4]. Compute the conditional probability that X lies between 2 and 3 given that X lies between 1 and 3, i.e. compute $\Pr(2 < X < 3 | 1 < X < 3)$.

(A) 1/2	(B) 2/3	(C) 1/8	(D) 1/3
(E) 3/4	(F) 5/8	(G) 1/4	(H) 3/8

12. The number of clicks of a Geiger counter is a Poisson process with a mean of 1 click per minute. What is the probability there are four or more clicks per 3 minutes?

(A) $1 - 2e^{-1}$	(B) $1 - 13e^{-3}$	(C) $12e^{-3}$	(D) $1 - 21e^{-4}$
(E) $2e^{-1}$	(F) $1 - 5e^{-3}$	(G) $1 - 12e^{-3}$	(H) $21e^{-4}$

13. The random variables X and Y are independent exponentially distributed random variables with mean waiting time of 0.2 seconds and 0.5 seconds, respectively. Compute the probability that Y occurs before X, $\Pr(Y < X)$. To get credit you must set up and evaluate the integral.

(A) 2/5	(B) 3/5	(C) 2/7	(D) 2/3
(E) 4/5	(F) 5/7	(G) 7/10	(H) 7/8

14. If R is an invertible 3×3 matrix such that $R \begin{bmatrix} \text{a} \\ \text{b} \\ \text{c} \end{bmatrix} = \begin{bmatrix} \text{b} \\ \text{-a} \\ \text{c} \end{bmatrix}$, then compute $R^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(A) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ (C) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

(E) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ (F) $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ (G) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ (H) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

15. Find the value of the constant k for which the following system of linear equations

$$\begin{cases} x + 2y + z = 5 \\ x - y + z = 1 \\ 2x + y - kz = 3 \end{cases}$$

has exactly one solution.

(A) $k = -3$	(B) $k \neq 2$	(C) $k = 2$	(D) $k \neq -3$
(E) $k \neq 3$	(F) $k = 3$	(G) $k = -2$	(H) $k \neq -2$

16. We know $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix}$. Find $A = \begin{bmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{bmatrix}$ and compute the sum of its entries $S = a + b + c + d$.

(A) $5/2$ (B) $-7/2$ (C) 4 (D) 7

(E) $15/2$ (F) 5 (G) -6 (H) $9/2$

17. Ann, Bill and Charlie are playing catch. Ann throws to Bill and Charlie with equal probability 1/2 and Charlie throws to Ann and Bill also with equal probability 1/2, but Bill throws to Ann 2/3 of the time and to Charlie 1/3 of the time. What is the probability that Bill will have the ball in the long run?

(A) 8/27	(B) 1/3	(C) 10/27	(D) 2/9
(E) 1/2	(F) 7/9	(G) 2/3	(H) 5/18

18. A company is divided into two divisions, I and II. To produce a \$1 worth of product in division I requires 40 cents spent in that division and 20 cents spent in division II. To produce a \$1 worth of product in division II requires 10 cents spent in division I and 30 cents spent in division II. How should the production levels be set in order to meet a demand for \$6 million worth of product I and \$10 million worth of product II? The production level in millions=

(A) $\begin{bmatrix} 14 \\ 16 \end{bmatrix}$ (B) $\begin{bmatrix} 11 \\ 16 \end{bmatrix}$ (C) $\begin{bmatrix} 8 \\ 12 \end{bmatrix}$ (D) $\begin{bmatrix} 6 \\ 10 \end{bmatrix}$

(E) $\begin{bmatrix} 16 \\ 21 \end{bmatrix}$ (F) $\begin{bmatrix} 13 \\ 18 \end{bmatrix}$ (G) $\begin{bmatrix} 16 \\ 18 \end{bmatrix}$ (H) $\begin{bmatrix} 13 \\ 16 \end{bmatrix}$

19. Two rods A and B are to be welded end to end to make 3 meter rod. The lengths of each of the rods is a normal distributed random variable with means μ and standard deviations σ given in the table below. What is the probability that the welded rods will be within 1 centimeter of 3 meters, i.e. $\Pr(|X_A + X_B - 3 \text{ m}| < 1 \text{ cm})$. Note 1 centimeter $= 10^{-2}$ meters.

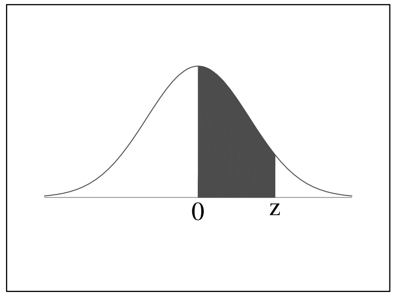
$$\text{A} : \mu = 1 \text{ meters}, \sigma = 1 \text{ centimeters}$$

$$\text{B} : \mu = 2 \text{ meters}, \sigma = \sqrt{3} \text{ centimeters}$$

Circle the closest answer. Indicate what you looked up and how you used it.

(A) 95%	(B) 80%	(C) 75%	(D) 50%
(E) 40%	(F) 25%	(G) 15%	(H) 5%

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Answer key:
DCCG EHDA EGAB CFHD BFE