

## Math 115

### Final Exam – Spring 2009

May 7, 2009

1. Evaluate  $\int_0^3 \int_{\sqrt{x}}^1 e^{y^3} dy dx$ .

- (A) 1            (B)  $e$             (C)  $1 - e$             (D)  $\frac{e-1}{2}$   
(E)  $e-1$             (F)  $\frac{1}{2}e-1$             (G)  $\sqrt{e}$             (H) None of these
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2. Find the  $z$ -coordinate of the point on the plane  $x + 2y + 3z = 13$  closest to the point  $(1, 1, 1)$ .

- (A) 0            (B) 2            (C)  $\frac{5}{2}$             (D)  $\frac{7}{4}$   
(E)  $\frac{3}{2}$             (F)  $\frac{3}{4}$             (G)  $\frac{7}{2}$             (H) None of these
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3. The packaging department in a company has been asked to design a rectangular box with no top and a partition down the middle. The box must have a volume of 48 cubic inches. Find the dimensions of the box that will minimize the material used to construct the box. The sum of these dimensions is:

- (A) 22            (B) 18            (C) 13            (D) 11  
(E) 20            (F) 16            (G) 12            (H) None of these
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4. Consider the surface  $x^2 + 2y^2 - 3z^2 = 3$ . Find the tangent plane to the surface at the point  $(2, -1, 1)$ . Find where this plane intersect the  $y$ -axis.

This plane intersects the  $y$ -axis at  $y =$

- (A) 0            (B) 1            (C) -1            (D)  $-\frac{2}{3}$   
(E)  $\frac{3}{2}$             (F)  $-\frac{3}{2}$             (G)  $\frac{2}{3}$             (H) None of these
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5. Let  $f(x, y) = \sin(2x + y)$ . Find the value of the partial derivative  $f_{xy}\left(\pi, \frac{\pi}{2}\right)$ .

- (A)  $\sqrt{2}$             (B)  $2\sqrt{2}$             (C) 0            (D) -2  
(E)  $-\sqrt{2}$             (F)  $-2\sqrt{2}$             (G) 2            (H) None of these
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6. A pair of fair dice is thrown. If the two numbers appearing upturned are different, what is the probability that the sum is 8?

- A)  $\frac{5}{6}$  B)  $\frac{4}{15}$  C)  $\frac{1}{9}$  D)  $\frac{5}{36}$  E)  $\frac{1}{6}$  F)  $\frac{2}{15}$  G)  $\frac{1}{5}$  H) None of these
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7. A fair coin is tossed 4 times. Let  $X$  be the random variable which denotes the longest string of heads. What is the expected value of  $X$ ?

- A)  $\frac{25}{16}$  B)  $\frac{31}{16}$  C)  $\frac{29}{16}$  D)  $\frac{33}{16}$  E)  $\frac{19}{16}$  F)  $\frac{41}{32}$  G)  $\frac{27}{16}$  H) None of these
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8. Box A contains nine cards numbered 1 through 9, and Box B contains five cards numbered 1 through 5. A box is chosen at random and a card drawn;

- if the card shows an even number, another card is drawn from the same box
- if the card shows an odd number, a card is drawn from the other box

What is the probability that both cards are even?

- A)  $\frac{1}{12}$  B)  $\frac{1}{6}$  C)  $\frac{1}{20}$  D)  $\frac{1}{3}$  E)  $\frac{2}{15}$  F)  $\frac{4}{15}$  G)  $\frac{362}{2025}$  H) None of these
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9. In the game of Parcheesi<sup>®</sup> each player rolls a pair of dice on each turn. In order to begin the game, you must throw a 5 on at least one of the dice, or a total of 5 on the two dice. What is the probability that you can begin the game on your first turn?

- A)  $\frac{5}{12}$  B)  $\frac{4}{9}$  C)  $\frac{11}{36}$  D)  $\frac{1}{9}$  E)  $\frac{2}{3}$  F)  $\frac{13}{36}$  G)  $\frac{1}{2}$  H) None of these
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10. The joint probability distribution function for  $X$  and  $Y$  where  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$

is  $f(x, y) = \frac{2}{3}(x + 2y)$ . Find the  $P(Y > 2X)$ .

- A)  $\frac{2}{3}$  B)  $\frac{1}{4}$  C)  $\frac{25}{32}$  D)  $\frac{3}{5}$  E)  $\frac{1}{2}$  F)  $\frac{2}{5}$  G)  $\frac{3}{4}$  H) None of these
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11. Consider the matrix  $A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{pmatrix}$ . Find  $A^{-1}$ . The sum of the entries in the

first row of  $A^{-1}$  is

- (A) 0            (B) -2            (C) -17            (D) -3  
(E) 1            (F) -23            (G) -6            (H)  $A$  is singular.
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12. An auto insurance company classifies its customers in 3 categories: poor, satisfactory, and preferred. Each year, 40% of those in the poor category are moved to the satisfactory category and 20% of those in the satisfactory category are moved to the preferred. Also, 40% in the preferred category are moved to the satisfactory category, and 20% in the satisfactory category are moved to the poor category. Customers are never moved from poor to preferred or preferred to poor in a single year. Assuming that these percentages remain valid over a long period of time, how many customers can the company expect to have in the satisfactory category in the long run?

- (A) 0            (B) 50%            (C) 75%            (D) 65%  
(E) 10%            (F) 25%            (G) 60%            (H) None of these
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13. For the opening night at the Opera House, a total of 1000 tickets were sold. Front orchestra seats cost \$80 each, rear orchestra seats cost \$60 each, and front balcony seats cost \$50 each. The total receipts for the performance were \$62,800. The combined number of tickets sold for the front orchestra and rear orchestra exceeded twice the number of front balcony tickets sold by 400. Determine how many tickets were sold for the front orchestra seats.

- (A) 100            (B) 140            (C) 160            (D) 320  
(E) 200            (F) 240            (G) 260            (H) None of these
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14. Four rods A, B, C and D are to be welded end to end to make a 45 centimeter rod. The length of each of the rods is a normally distributed random variable with means  $\mu$  and standard deviations  $\sigma$  given in the table below. What is the probability that the assembled rods will be within 1 centimeter of 45 centimeters?

In other words,  $\Pr(|X_A + X_B + X_C + X_D - 45| < 1)$

A:  $\mu = 10\text{cm}, \sigma = 1\text{cm}$

B:  $\mu = 11\text{cm}, \sigma = 2\text{cm}$

C:  $\mu = 12\text{cm}, \sigma = 2\text{cm}$

D:  $\mu = 13\text{cm}, \sigma = 4\text{cm}$

- (A) 0.0793   (B) 0.1554   (C) 0.3446   (D) 0.8446  
 (E) 0.1586   (F) 0.0875   (G) 0.6554   (H) None of these
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Do **ONLY ONE** of the following two questions.

15. A simple economy consists of two sectors, agriculture and manufacturing. The input-output matrix is  $A = \begin{pmatrix} .4 & .1 \\ .2 & .3 \end{pmatrix}$ . How many units (in the form  $\begin{bmatrix} \text{agric.} \\ \text{manuf.} \end{bmatrix}$ ) should be produced by each sector to meet the consumer demand of 24 units of agriculture and 40 units of manufacturing?

- (A)  $\begin{bmatrix} 44 \\ 64 \end{bmatrix}$    (B)  $\begin{bmatrix} 56 \\ 78 \end{bmatrix}$    (C)  $\begin{bmatrix} 50 \\ 75 \end{bmatrix}$    (D)  $\begin{bmatrix} 32 \\ 48 \end{bmatrix}$   
 (E)  $\begin{bmatrix} 68 \\ 84 \end{bmatrix}$    (F)  $\begin{bmatrix} 62 \\ 82 \end{bmatrix}$    (G)  $\begin{bmatrix} 64 \\ 74 \end{bmatrix}$    (H)  $\begin{bmatrix} 52 \\ 72 \end{bmatrix}$

16. A fair coin is tossed 100 times. The probability that the number of heads is between 55 and 60, inclusive, is closest to:

- (A) 10.70%   (B) 11.78%   (C) 13.59%   (D) 15.54%  
 (E) 16.62%   (F) 83.38%   (G) 86.41%   (H) 89.30%
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**ANSWERS:**

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|------|-------|-------|
| 1. E | 6. F  | 11. D |
| 2. C | 7. G  | 12. B |
| 3. G | 8. E  | 13. F |
| 4. F | 9. A  | 14. B |
| 5. D | 10. B | 15. H |
|      |       | 16. C |

