Math 115 **Final Exam** **Fall 2005**

ANSWERS AT THE END

1. X is a continuous random variable on the interval [0,1] whose density function is of the form kx^2 for some constant k. What is Var(X)?

   A. 1/80 B. 1/40 C. 3/80 D. 1/20

   E. 5/80 F. 3/40 G. $\sqrt{\frac{1}{80}}$ H. $\sqrt{\frac{3}{80}}$

2. A Geiger counter clicks, on the average, every 15 seconds. (The number of clicks is a Poisson process.) Let X be the total number of clicks of three such geiger counters in a minute. How many clicks per minute is one standard deviation of X.

   A. 12 B. 45 C. $\sqrt{45}$ D. 8
   E. $\sqrt{12}$ F. 6 G. $\sqrt{6}$ H. $\sqrt{8}$

3. Find $\dfrac{\partial^2}{\partial x \partial y}\left(\dfrac{x}{y} + y^2 + x^y\right)$

   A. $\ln(y)+x^y \ln(x)$ B. $-\frac{1}{y^2}+x^y \ln(x)$ C. $-\frac{1}{y^2}+x^{y-1}[y \ln(x)+1]$
   D. $x \ln(y) + x^{y-1} ln(x)$ E. $\frac{x}{y^3}+x^{y-1}[y \ln(x)+1]+2$ F. $-\frac{1}{y^2}+x^{y-1}$
   G. $\frac{2}{y^2}+x^y \ln(x)$ H. 0

4. If the measurements of a and b to the nearest 1/10 of an inch are $a = 5$ inches and $b = 6$ inches then the maximum percentage error in calculating the area $A = \pi ab$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is closest to:

   A. $30\pi\%$ B. $1.1\pi\%$ C. $18\pi\%$ D. $11\pi\%$
   E. $3\pi\%$ F. 61/9600% G. 10/3% H. 11/3%

5. The waiting time for an elevator is an exponentially distributed random variable with mean 3 minutes. When your guest arrives at your floor you ask if he had to wait longer than 6 minutes for the elevator. He says no. What is the probability that he had to wait at least 3 minutes?

   A. 1/2 B. $(e-1)/(e-2)$ C. $(e^{-1})/(e^{-2})$
   D. $(e^{-2})/(e^{-1})$ E. e^{-2} F. $e^{-1}-e^{-2}$
   G. $(e^{-1}-e^{-2})/(e^{-2}-1)$ H. $(e^{-1}-e^{-2})/(1-e^{-2})$

6. Suppose that a random variable X is uniformly distributed on the interval $[1, 6]$. The expected value of $1/(X+1)$ is:

A. $1/7$ B. $1/5$ C. $\ln(7/2)/5$ D. $\ln(7/2)/6$
E. $\ln(7)/5$ F. $5/2$ G. $7/2$ H. $1/\ln(7/2)$

7. Let $w = \ln(1 + \frac{x^2}{2}) - \arctan(x)$ and $x = 3e^u \cos(v) + v$. Find $\frac{\partial w}{\partial v}$ at $u = v = 0$.

A. $147/110$ B. $196/110$ C. 0 D. $49/110$
E. $3/10$ F. 3 G. $4/10$ H. undefined

8. The tangent plane to the surface $x^2 - 2y^2 + z^2 + yz = 2$ at the point $(2, 1, -1)$ intersects the z–axis at the point:

A. $(0, 0, -4)$ B. $(0, 0, 4)$ C. $(0, 0, -1)$ D. $(0, 0, 3)$
E. $(0, 0, -3)$ F. $(0, 0, 2)$ G. $(0, 0, -2)$ H. $(0, 0, \sqrt{3})$

9. Given that

$$A^2 = \begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad A^3 = \begin{pmatrix} 1 & -2 \\ 2 & -5 \end{pmatrix} \quad \text{find} \quad A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and compute the product $P = ab$. Then $P =$

A. 2 B. -2 C. 1 D. -1
E. 0 F. 3 G. -3 H. 4

10. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

A. $\sqrt{6}$ B. $2\sqrt{6}$ C. 2 D. $5\sqrt{6}/6$
E. 1 F. $\sqrt{6}/6$ G. $\sqrt{6}/3$ H. $2\sqrt{3}$

11.

$$\int_1^4 \int_{x-1}^3 e^{y^2} dy dx =$$

A. 0. B. 1. C. e. D. $(e^9 - 1)/2$.
E. $e^{16} - 1$. F. $(e^{16} - e)/2$. G. $e^9 - 1$. H. $e^{16} - e$.

12. Which of the following statements regarding the system of equations

$$\begin{cases} x - y + z = 1 \\ 3x - 3y + 3z = 3 \\ 2x + 4y + 6z = k \end{cases}$$

is true ?

A. The system has a unique solution for any value of k.
B. The system has a unique solution only when $k = 8$.
C. The system has a unique solution only when $k = 0$.
D. The system only has infinitely many solutions when $k = 8$.
E. The system infinitely many solutions for every value of k.
F. The system has a unique solution for $k = 0$ and infinitely many when $k = 8$.
G. The system never has a solution.
H. The system has infinitely many solutions when $k > 0$, a unique solution when $k = 0$, and no solutions for $k < 0$.

13. $A = \begin{bmatrix} 0 & 3 & -2 \\ 2 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

The sum of the entries in the third row of A^{-1} equals:

A. -2 B. 0 C. 2 D. 4
E. 8 F. 10 G. -4 H. -8

14. A chest of drawers has three drawers: the top, the middle, and the bottom. The top drawer contains 3 black and 2 white pairs of socks, the middle drawer has 2 black and 4 white pairs of socks, and the bottom drawer has 4 black and 2 white pairs of socks. A drawer is selected at random (any drawer is equally likely to be chosen) and a pair of socks from the drawer is chosen at random. Given that the pair is white, what is the probability that it came from the middle drawer?

A. $1/8$ B. $2/9$ C. $7/15$ D. $1/2$
E. $2/15$ F. $10/21$ G. $15/32$ H. $5/9$

15. A bridge hand consists of 13 card from a standard 52-card deck. Find the probability that a bridge hand contains all four aces?

A. $\frac{1}{13}$ B. $\frac{4}{13}$ C. $\frac{48!}{9!}\frac{13!}{52!}$ D. 0
E. $\frac{4!}{52!}$ F. $1 - \frac{13!}{52!}$ G. $\frac{3}{4}$ H. $\frac{48!}{52!}$

16. Consider three events E, F and G. Assume that they have the following probabilities: $Pr(E) = \frac{1}{2}$, $Pr(F) = \frac{9}{20}$, and $Pr(G) = \frac{2}{5}$. Assume that we also know the following conditional probabilities $Pr(E|F) = \frac{2}{3}$, $Pr(E|G) = \frac{5}{8}$, $Pr(F|G) = \frac{3}{8}$, and $Pr((E \cap G)|F) = \frac{2}{9}$. Then $Pr(F \cup (E \cap G))$ is (There may be more information than is needed to solve the problem):

A. 1/4 B. 1/10 C. 16/25 D. 3/5
E. 2/9 F. 7/10 G. 3/4 H. 19/20

17. The SAT score of students in a high school has a normal distribution with mean 1000 and standard deviation 100. In order to be admitted to Prestigious University, the SAT score has to be at least 1200. If the high school has 1200 students, and they all applied to PU, the number of students expected to be admitted lies in the range

A. $171 - 200$ B. $91 - 101$ C. $80 - 91$ D. $61 - 80$
E. $31 - 60$ F. $151 - 171$ G. $102 - 150$ H. $0 - 30$

18. An economy depends on two basic products: coal and gas. In order to produce 1 unit of coal, one needs 0.4 units of coal and 0.5 units of gas. In order to produce 1 unit of gas, one needs 0.2 units of coal and 0.6 units of gas. What should the production be in order to satisfy a final demand of 2 million units of coal and 4 million units of gas ?

A. 7.6 million units of coal and 3.9 million units of gas.
B. 15.2 million units of coal and 8.33 million units of gas.
C. 11.43 million units of coal and 24.29 million units of gas.
D. 13.9 million units of coal and 24.29 million units of gas.
E. 17.1 million units of coal and 4.33 million units of gas.
F. 13.2 million units of coal and 24.29 million units of gas.
G. 3.3 million units of coal and 8.33 million units of gas.
H. 11.3 million units of coal and 8.33 million units of gas.

ANSWERS ON THE NEXT PAGE

Answers

1. C. $3/80$
2. E. $\sqrt{12}$
3. C. $-\frac{1}{y^2} + x^{y-1}(y\ln(x)+1)$
4. H. $11/3$
5. H. $(e^{-1}-e^{-2})/(1-e^{-2})$
6. C. $\ln(7/2)/5$
7. D. $49/110$
8. A. $(0,0,-4)$
9. B. -2
10. D. $5\sqrt{6}/6$
11. D. $(e^9-1)/2$.
12. E. The system infinitely many solutions for every value of k.
13. F. 10
14. F. $10/21$
15. C. $\frac{48!}{9!}\frac{13!}{52!}$
16. D. $3/5$
17. H. $0-30$
18. C. 11.43 million units of coal and 24.29 million units of gas.

P(0 <z< a)

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990