Math 115 **Final Exam** **Fall 2004**

ANSWERS AT THE END

1. The tangent plane to the ellipsoid $x^2/4 + y^2 + z^2/9 = 3$ at the point $(-2, 1, -3)$ intersects the x–axis at the point:

   A. $(4, 0, 0)$ B. $(3, 0, 0)$ C. $(-3, 0, 0)$ D. $(2, 0, 0)$
   E. $(6, 0, 0)$ F. $(-6, 0, 0)$ G. $(1, 0, 0)$ H. $(-1, 0, 0)$

2. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

   A. $\sqrt{6}$ B. $2\sqrt{6}$ C. 2 D. $5\sqrt{6}/6$
   E. 1 F. $\sqrt{6}/6$ G. $\sqrt{6}/3$ H. $2\sqrt{3}$

3. The maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $0 \le x \le 3, 0 \le y \le 2$ are:

   A. $max = 9, min = 1$
   B. $max = 9, min = 0$
   C. $max = 1, min = 0$
   D. $max = 4, min = 0$
   E. $max = 4, min = 1$
   F. $max = 9, min = 4$
   G. $max = 1, min = -1$
   H. $max = 9, min = -1$

4. If $z = f(x, y)$ where f is differentiable, $x = g(t)$, $y = h(t)$, $g(3) = 2$, $g'(3) = 5, h(3) = 7, h'(3) = -4$, $\frac{\partial f}{\partial x}(2, 7) = 6$ and $\frac{\partial f}{\partial y}(2, 7) = -8$, then dz/dt when $t = 3$ is:

   A. 30 B. 32 C. 62 D. -30
   E. -30 F. -62 G. 2 H. 12

5. Given that

$$A^2 = \begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad A^3 = \begin{pmatrix} 1 & -2 \\ 2 & -5 \end{pmatrix} \quad \text{find} \quad A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and compute the product $P = ab$. Then $P =$

A. 2 B. -2 C. 1 D. -1 E. 0 F. 3
G. -3 H. 4

6. Each year, 1/10 of the people outside California move in, and 2/10 of the people inside California move out. If initially, 3/10 of the total population lives in California, in the long run, the fraction of the total population living in California will be:

A. 2/3 B. 1/3 C. 1/2 D. 4/5 E. 1/5 F. 2/5
G. 9/10 H. 1/8

7. Which of the following statements regarding the system of equations

$$\begin{cases} x - y + z = 1 \\ x + 2y + 3z = 4 \\ 2x + 4y + 6z = k \end{cases}$$

is true ?

A. The system has a unique solution for any value of k.
B. The system has a unique solution only when $k = 8$.
C. The system has a unique solution only when $k = 0$.
D. The system only has infinitely many solutions when $k = 8$.
E. The system infinitely many solutions for every value of k.
F. The system has a unique solution for $k = 0$ and infinitely many when $k = 8$.
G. The system never has a solution.
H. The system has infinitely many solutions when $k > 0$, a unique solution when $k = 0$, and no solutions for $k < 0$.

8. The SAT score of students in a high school has a normal distribution with mean 1000 and standard deviation 200. In order to be admitted to Prestigious University, the SAT score has to be at least 1300. If the high school has 1500 students, and they all applied to PU, the number of students expected to be admitted lies in the range

A. $171 - 200$ B. $91 - 101$ C. $80 - 91$ D. $61 - 80$ E. $31 - 60$
F. $151 - 171$ G. $102 - 150$ H. $0 - 30$

9. In a lottery, the players select 6 numbers between 1 and 49. What is the probability that the winning combination consists of all odd numbers ?

A. $C_{25,6}/P_{25,6}$ B. $C_{24,6}/P_{25,6}$ C. $25/49$ D. $1/2$ E. $C_{24,6}/C_{49,6}$
F. $C_{25,6}/C_{49,6}$ G. $C_{49,2}/C_{49,6}$ H. $P_{49,2}/C_{49,6}$

10. A class consists of 5 women and 4 men. If a committee of 5 is chosen at random, what is the probability that the women will outnumber the men ?

A. $1/7$ B. $9/14$ C. $3/14$ D. $5/14$ E. $2/7$ F. $1/2$
G. $1/3$ H. $4/5$

11. Suppose that a random variable X is uniformly distributed on the interval $[1, 6]$. The expected value of $1/X$ is:

A. $\ln(6)$ B. $1/5$ C. $\ln(6)/5$ D. $\ln(6)/6$ E. $\ln(5)/6$
F. $1/6$ G. $1/\ln(6)$ H. 0

12. An economy depends on two basic products: coal and gas. In order to produce 1 unit of coal, one needs 0.5 units of coal and 0.25 units of gas. In order to produce 1 unit of gas, one needs 0.4 units of coal and 0.2 units of gas. What should the production be in order to satisfy a final demand of 2 million units of coal and 4 million units of gas ?

A. 9.3 million units of coal and 2 million units of gas.
B. 10.67 million units of coal and 8.33 million units of gas.
C. 8.67 million units of coal and 6.33 million units of gas.
D. 5.3 million units of coal and 6.33 million units of gas.
E. 10.67 million units of coal and 4.33 million units of gas.
F. 5.3 million units of coal and 8.33 million units of gas.
G. 3.3 million units of coal and 8.33 million units of gas.
H. 11.3 million units of coal and 8.33 million units of gas.

13. Consider the function $f(x,y) = x^2 + 2xy - y^2 + 2x - 6y + 7$. Find its critical points and determine their type.

A. Local max at $(0,0)$.
B. Local min at $(0,0)$.
C. Saddle point at $(0,0)$.
D. Local max at $(1,-2)$.
E. Local min at $(1,-2)$.
F. Saddle point at $(1,-2)$.
G. There are no critical points.
H. There is more than one critical point.

14.
$$\int_1^3 \int_{x-1}^2 e^{y^2} dy dx =$$

A. 0. B. 1. C. e. D. $(e^9 - e)/2$.
E. $e^9 - 1$. F. $(e^4 - 1)/2$. G. $e^4 - 1$. H. $e^9 - e$.

15. Let the bivariate distribution of two continuous random variables X and Y be given by the joint probability density function:
$$f(x,y) = \begin{cases} \frac{1}{8}(x+y) & 0 \le x \le 2, 0 \le y \le 2 \\ 0 & otherwise \end{cases} \quad (1)$$

Then $Pr(X \ge Y | X \le 1)$ is

A. 0. B. 1. C. 3/8. D. 1/16. E. 1/6. F. 1/2.
G. 1/3. H. 2/3.

16. Cars, taxis, and buses pass through a certain intersection. It is noticed that 70% are cars, 20% are taxis, and 10% are buses. It is also noticed that 40% of the cars turn right while 20% turn left, 10% of the taxis turn right while 60% turn left, and of the buses 80% turn right while 10% turn left.

If a vehicle goes straight what is the probability that it was a car?

A. 0. B. 1/5. C. 2/5. D. 3/5. E. 4/5.
F. 2/25. G. 3/25. H. 4/25.

17. A Geiger counter clicks, on the average, every 30 seconds. (The number of clicks is a Poisson process.) It is known that during one particular minute the counter clicked at least 2 times. What is the probability that the number of clicks during that minute was at most 3?

A. $\frac{10}{3}$ B. $\frac{10}{3}e^{-2}$ C. $3e^{-2}$ D. $1-3e^{-2}$
E. $\frac{1}{2}$ F. $\frac{10}{9}e^{-2}$ G. $\frac{1-3e^{-2}}{\frac{10}{3}e^{-2}}$ H. $\frac{\frac{10}{3}e^{-2}}{1-3e^{-2}}$

18. If X is a random variable on the interval $[0,1]$ with the cumulative distribution function $F(x) = 2x - x^2$, what is its variance?

A. 1/6. B. 1/2. C. 1/18. D. 2/3. E. 0. F. 1/3.
G. 1/9. H. 13/180.

Answers

1. F. $(-6,0,0)$
2. D. $5\sqrt{6}/6$
3. B. $\max = 9,\ \min = 0$
4. C. 62
5. B. -2
6. B. 1/3
7. D. The system only has infinitely many solutions when $k = 8$
8. B. $91 - 101$
9. F. $C_{25,6}/C_{49,6}$
10. B. 9/14
11. C. $\ln(6)/5$
12. B. 10.67 million units of coal and 8.33 million units of gas.
13. F. Saddle point at $(1,-2)$.
14. F. $(e^4-1)/2$.
15. E. 1/6.
16. E. 4/5.
17. H. $\frac{\frac{10}{3}e^{-2}}{1-3e^{-2}}$
18. C. 1/18.

P(0 <z< a)

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990