

Name: _____
Penn ID: _____
Recitation & TA: _____

FINAL EXAM
Math 115, Fall 2003

December 18, 2003

SCORE	
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No calculators. You must show work to get credit. Please put your answer in the space provided. You are permitted a sheet of notes and a table of the normal distribution.

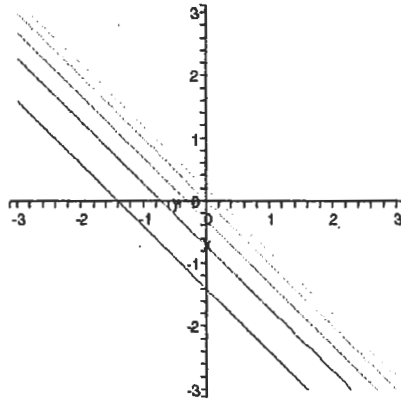
1. Use the linear approximation of $f(x, y) = \sqrt{4x^2 + y^2}$ at $(2, 3)$ to approximate $\sqrt{4(2.02)^2 + (2.97)^2}$: _____

- a. 5.014
- b. 5.018

- c. 5.032
- d. 5.05

- e. 5.064
- f. 5.082

2. Which of the following functions' contours are drawn in the graph?: _____



- a. $x - y$
- b. $e^x e^y$
- c. ye^x

- d. $e^x + e^y$
- e. $x + 4y$
- f. None of these

3. Consider the function $g(x, y) = x^3 + y^2 - 3x + 2y$. Find the value of the function at the saddle point.: _____

- a. -5
- b. -3
- c. -1

- d. 1
- e. 3
- f. 5

4. Find the minimum value of $f(x, y) = x^2 + y^2 - 2x + 4$ on the circle $x^2 + y^2 = 1$: _____

- a. 2
- b. 3
- c. 4

- d. 5
- e. 7
- f. None of these

5. Evaluate the double integral $\iint_R x dA$, where R is the region bounded by the curves $y = x$ and $y = x^2$: _____

a. $-1/22$

b. $-1/20$

c. 0

d. $1/20$

e. $1/12$

f. 1

6. Find the x -intercept (the value of x where the object intersects the x -axis) of the tangent plane to the surface $z = \frac{y}{x}$ at the point $(2, 4, 2)$: _____

a. -6

b. -4

c. -2

d. 0

e. 2

f. 4

7. A committee of 3 politicians is chosen at random from a group of 2 republicans, 2 democrats, and 1 libertarian. Find the probability that no other party has more representatives than the democrats: _____

- a. $1/10$
- b. $1/5$
- c. $3/10$

- d. $2/5$
- e. $1/2$
- f. $7/10$

8. A pair of dice (with sides numbered 1 through 6) are rolled 10 times. For each roll of the dice the sum of the dice is noted. Suppose that the dice are rolled ten times. Find the probability that exactly twice a "2", "7", or "11" is rolled.: _____

- a. $\frac{45 \cdot 5^8}{6^{10}}$
- b. $\frac{9}{4^{10}}$
- c. $\frac{3^8}{4^{10}}$

- d. $\frac{45 \cdot 9}{4^{10}}$
- e. $\frac{45 \cdot 3^8}{4^{10}}$
- f. None of these

9. Find the expected value of the continuous random variable X with the probability density function $f(x) = \begin{cases} 3x^2 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \therefore$ _____

- a. 1/4
- b. 1/3
- c. 1/2

- d. 3/4
- e. 4/5
- f. 1

10. Students in a large class take two exams whose scores are approximately normally distributed. The first exam has a mean of 75 and a standard deviation of 3. The second exam has a mean of 85 with a standard deviation of 4. Find the *approximate* probability that the sum of the two scores for a student chosen at random will be larger than 170.: _____

- a. 1%
- b. 2%
- c. 3%

- d. 4%
- e. 5%
- f. 10%

11. The random variables X, Y have the joint probability density function

$$f(x, y) = \begin{cases} x + y & \text{for } 0 \leq x, y \leq 1 \\ 0 & \text{otherwise} \end{cases}. \text{ Find } \Pr(2X > Y): \underline{\hspace{2cm}}$$

- a. $5/24$
- b. $1/4$
- c. $3/8$

- d. $1/2$
- e. $3/4$
- f. $19/24$

12. A lottery consists of choosing 7 different numbered balls, the winning numbers, at random from a group of 50. A player chooses 7 numbers on any one ticket. Find the *approximate* expected number of winning numbers chosen on a ticket.:

- a. 0.5
- b. .75
- c. 1

- d. 1.5
- e. 2
- f. 3

13. Suppose calls coming into any particular telephone line at a telephone exchange come in at an average rate of 3 per minute are governed by a Poisson process. Find the probability that no phone calls come in on 3 different lines coming into the exchange in one minute.: _____

- a. e^{-9}
- b. e^{-3}
- c. $4e^{-3}$

- d. e^{-1}
- e. $1 - e^{-3}$
- f. $1 - e^{-9}$

14. The waiting times for a single telephone line have an exponential distribution with a mean of $1/3$ minute. Find the probability that no calls arrive on a single line in the first two minutes of a shift.: _____

- a. e^{-6}
- b. e^{-3}
- c. $e^{-1/3}$

- d. $e^{-1/6}$
- e. $1 - e^{-1/6}$
- f. $1 - e^{-3}$

15. Find the value for the constant k for which the following system of linear

equations $\left\{ \begin{array}{l} x + 2y + z = 10 \\ x + y + z = 4 \\ 2x + 3y + kz = 2 \end{array} \right\}$ has no solution.: _____

- a. -2
- b. -1
- c. 0

- d. 1
- e. 2
- f. No such k exists

16. A large house can hold at most 5 rabbits or guinea pigs. Rabbits eat 2 bags of rodent food per month and while guinea pigs eat only 3 bags. The owner of the house will only buy 12 bags per month. Rabbits provide 5 units of joy each and guinea pigs provide 2 units of joy each. Find the maximum amount of joy that the rabbits and guinea pigs can bring, given the constraints.: _____

- a. 0
- b. 8
- c. 10

- d. 19
- e. 25
- f. 30

17. From one week to the next, a student will get credit for their homework 10% of the time after not getting credit the week before, and the student will get credit for their homework 50% of the time after getting credit for the homework. In the long run, what is the probability that the student will get credit for the weekly homework?: _____

- a. $1/9$
- b. $1/6$
- c. $1/3$

- d. $1/2$
- e. $5/6$
- f. $8/9$

18. Find the sum of the entries of the 2×2 matrix which is equal to the following

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}: \text{_____}$$

- a. -4
- b. -2
- c. -1

- d. 0
- e. 1
- f. 2