MATH 115 FINAL EXAM DECEMBER 2002

1. Consider the surface $z = f(x, y) = 2x^2 + y^2$. Find the tangent plane to the surface at the point $(x, y, z) = (1, 1, 3)$ and find where this plane intersects the z-azis. Plane intersects the z-axis at $z =$

$$A.3 \quad B.2 \quad C.0 \quad D.-3 \quad E.-4 \quad F.-6 \quad G.-8 \quad H.-10$$

2. If $z = f(x, y)$ is implicitly defined by the equation $x^4 + y^2 + z^2 = 14$ in a neighborhood of $(x, y, z) = (1, 2, 3)$. Find $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$ at this point.

$$A.-4/3 \quad B.3/2 \quad C.0 \quad D.-1 \quad E.-5/2 \quad F.2 \quad G.-2 \quad H.-5/4$$

3. Let $r(x, y) = \sqrt{x^2 + y^2}$. Note $r(3, 4) = 5$. Using differentials to approximate $r(2.9, 4.1)$ one gets

$$A.5-\frac{1}{25} \quad B.5-\frac{1}{50} \quad C.5+\frac{1}{50} \quad D.5+\frac{1}{25} \quad E.5-\frac{1}{20} \quad F.5 \quad G.5+\frac{1}{30} \quad H.5-\frac{1}{80}$$

4. The function $f(x, y) = x^3 + y^2 - 3x + 2y$ has exactly one saddle point. The value of f at the saddle point is

$$A.0 \qquad B.1 \qquad C.-1 \qquad D.2 \qquad E.5 \qquad F.-2 \qquad G.3 \qquad H.-3$$

5. Find the shortest distance from point $(x, y) = (2, 0)$ and the curve $y^2 - x^2 = 4$.

$$A.\sqrt{3}/3 \quad B.\sqrt{2}/2 \quad C.1 \quad D.\sqrt{2} \quad E.\sqrt{3} \quad F.\sqrt{7} \quad G.\sqrt{6} \quad H.3$$

6.Evaluate

$$\int_0^1 \int_{y^2}^1 ye^{-x^2}\, dx\, dy$$

$$A.(e-1)/4 \; B.e^2-2 \; C.(1-e^{-1})/4 \; D.2\ln(2) \; E.0 \; F.4 \; G.(e-2)/e \; H.(e-1)/2$$

7. A fair coin is flipped 6 times. What is the probability that there are exactly 3 heads.

$$A.1/8 \quad B.3/16 \quad C.1/4 \quad D.5/16 \quad E.11/32 \quad F.3/8 \quad G.7/16 \quad H.1/2$$

8. A person draws two socks at random out of a drawer containing 2 black socks and 4 red socks. Given that the second sock drawn is red what is the probability that the first sock is black.

$$A.1/3 \quad B.2/3 \quad C.1/4 \quad D.1/2 \quad E.3/4 \quad F.2/5 \quad G.4/5 \quad H.3/7$$

9. A box contains ten apples, three of which are bruised. If six apples are chosen at ramdom from the box what is the probablity that at least two of the chosen apples are bruised?

$$A.\frac{11}{30} \quad B.\frac{19}{30} \quad C.\frac{13}{45} \quad D.\frac{25}{36} \quad E.\frac{3}{5} \quad F.\frac{2}{5} \quad G.\frac{3}{7} \quad H.\frac{2}{3}$$

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10. There are two dice numbered 1,1,2,2,3,3 so each die has probability of 1/3 of producing a 1, 2 or 3. The two die are tossed. Let X be the maximum number showing. What is the expected value of X? (Hint, calculate the probability that $X = 1$, 2 and 3.)

$A.5/2 \quad B.7/3 \quad C.9/4 \quad D.16/7 \quad E.18/7 \quad F.20/9 \quad G.22/9 \quad H.26/9$

11. Suppose X is a random variable taking values in the interval $[0, 2]$ with probability density function $f(x) = 1 - x/2$. What is the variance of X?

$A.0 \quad B.1/18 \quad C.1/9 \quad D.2/9 \quad E.1/3 \quad F.4/9 \quad G.1/2 \quad H.5/9$

12. A point (x, y) is chosen at random in the rectangle $0 \leq x \leq 3$ and $0 \leq y \leq 2$. Compute the probability that the absolute value of the difference between x and y is less than one. (i.e. Compute Prob($|x - y| < 1$).

$A.0 \quad B.1/4 \quad C.1/3 \quad D.2/5 \quad E.7/12 \quad F.5/12 \quad G.2/3 \quad H.3/4$

13. The number of clicks of a Geiger counter is a Poisson process with a mean of 2 clicks per minute. What is the probability there are three or more clicks in 2 minutes.

$A.2/e \quad B.1-2/e \quad C.1-5e^{-2} \quad D.1-3e^{-3} \quad F.1-13e^{-4} \quad G.12e^{-4} \quad H.1-21e^{-4}$

14. The random variables X and Y are exponentially distributed random variables with mean waiting times of 0.2 seconds and 0.5 seconds so (so X has as probability density function $f(t) = 5e^{-5t}$ and Y has a probability density function $g(t) = 2e^{-2t}$). The random variable X and Y are independent. Compute the probabily X occurs before Y. Prob $(X < Y)$. (To get credit you must set up and evaluate the integral.)

$A.2/5 \quad B.3/5 \quad C.5/7 \quad D.2/3 \quad E.4/5 \quad F.4/7 \quad G.7/10 \quad H.7/8$

15. Three people A, B and C are playing catch. The probabilities of each throwing to the others are: Prob(A$\rightarrow B$) = 1/4 Prob(A$\rightarrow C$) = 3/4 Prob(B$\rightarrow A$) = 1/4 Prob(B$\rightarrow C$) = 3/4 Prob(C$\rightarrow A$) = 1/2 Prob(C$\rightarrow B$) = 1/2. What is the probability that A will have the ball in the long run?

$A.4/9 \quad B.1/3 \quad C.2/7 \quad D.2/3 \quad E.1/2 \quad F.7/9 \quad G.2/9 \quad H.5/18$

16. For each of the systems of equations below indicate whether there are no solutions (0), a unique solution (1) or infinitely many solutions (∞)

$$I. \quad \begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ & & y & + & z & = & 1 \\ x & + & 2y & + & 2z & = & 2 \end{array}$$

$$II. \quad \begin{array}{ccccccccc} x & + & y & + & 2z & + & w & = & 1 \\ & & y & + & z & + & 2w & = & 1 \\ & & & & z & + & w & = & 4 \\ & & & & & & w & = & 3 \end{array}$$

$$III. \quad \begin{array}{rcrcrcl} x & + & y & + & z & = & 1 \\ & & y & + & z & = & 1 \\ x & + & 2y & + & 2z & = & 3 \end{array}$$

17. Given

$$A^2 = AA = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad A^3 = AAA = \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix} \quad \text{find} \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

and compute the sum of the coefficients $S = a + b + c + d$. The sum =

$A.2 \quad B.5 \quad C.0 \quad D.-1 \quad E.-2 \quad F.3/2 \quad G.-3/2 \quad H.1/2$

18. Apple pies take 12 ounces of filling and 4 ounces of dough to assemble and apple tarts take 2 ounces of filling and 2 ounces of dough. Apple pies cost \$7 each and apple tarts cost \$3. If there is 48 ounces of filling and 24 ounces of dough what is the maximum total income that can be made.

$A.\$24 \quad B.\$28 \quad C.\$30 \quad D.\$32 \quad E.\$34 \quad F.\$36 \quad G.\$39 \quad H.\$42$

19. Three rods A, B and C are to be welded end to end to make 5 meter rod. The lengths of each of the rods is a normal distributed random variable with means μ and standard deviations σ given in the table below. What is the probability that the assembled rods will be within 1 millimeters of 5 meters? (i.e. $\Pr(|X_A + X_B + X_C - 5m| < 1\text{mm})$ (1 millimeter = $10^{-3}meters$)

A	$\mu = 1$ meters	$\sigma = 1$ millimeters
B	$\mu = 2$ meters	$\sigma = 2$ millimeters
C	$\mu = 2$ meters	$\sigma = 2$ millimeters

Circle the closest answer. Indicate what you looked up and how you used it.

$A.95\% \quad B.80\% \quad C.75\% \quad D.50\% \quad E.40\% \quad F.25\% \quad G.15\% \quad H.5\%$

Answers.

1.D 2.A 3.C 4.B 5.G 6.C 7.D 8.F 9.H 10.G 11.D 12.E 13.F
14.C 15.C 16.$I.\infty$ $II.1$ $III.0$ 17.B 18.F 19.F