Math 114: Make-up Final Exam

Instructions:

1. Please sign your name and indicate the name of your instructor and your teaching assistant:

   A. Your Name:

   B. Your Instructor:

   C. Your Teaching Assistant:

2. This exam is 2 hours long and there are 20 questions.
3. You can use one handwritten two-sided page of notes; no books or calculators are allowed.
4. **It is important that you show your work for each problem.** To receive credit for a problem, you must both indicate the correct answer and show plausible work justifying your answer.
5. Do not come to the front of the class when the exam is over; we will pick up your exam from you.
6. **Don't take any work with you which is needed to justify your answers.**

The table below is for grading purposes only. You do not need to transfer your answers to this page.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.

Total:

(1) Find the length of the arc in $\mathbb{R}^3$ which is parameterized by the function

$$r(t) = (\frac{t^2}{2}, \frac{2\sqrt{2}t^{5/2}}{5}, \frac{t^3}{3})$$

for $0 \leq t \leq 1$.

(A) $(2^{3/2} - 1)/2$

(B) $(2^{3/2} - 1)/3$

(C) 1

(D) 6/5

(E) 5/6

(F) none of the above

Answer to 1:

(2) Find the area of the parallelogram in $\mathbb{R}^3$ which is spanned by the vectors $\vec{A} = \langle 1, 1, 1 \rangle$ and $\vec{B} = \langle 0, 2, 3 \rangle$.

(A) $\sqrt{14}$	(B) 4
(C) $\sqrt{12}$	(D) 5
(E) 3	(F) none of the above

Answer to 2:

(3) A guest on the Jerry Springer show attempts to evade one of the bouncers by following the path in the x-y plane described by $r(t) = (x(t), y(t)) = (t^3, t^2)$ for $-1 \leq t \leq 3$. What is the maximum length of the acceleration vector of the guest for $0 \leq t \leq 1$?

(A) 40.

(B) $\sqrt{40}$.

(C) 36.

(D) 6.

(E) none of the above.

Answer to 3:

(4) What is the angle between the planes $x + z = 0$ and $2x + 2y + z = 0$?

(A) 0 degrees

(B) 60 degrees

(C) 30 degrees

(D) 90 degrees

(E) 45 degrees

(F) none of the above.

Answer to 4:

(5) Find the distance between the point $(0, 0, 1)$ and the plane $x + 2y - z = 5$.

(A) 2

(B) 1

(C) 3

(D) 4

(E) $\sqrt{6}$

(F) none of above

Answer to 5:

(6) Does the limit $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{(x^2+y^2)}$ exist, and if it exists, what does it equal?

(A) Exists and equals 1.

(B) Exists and equals 0.

(C) The limit is not defined.

(D) Exists and equals 2.

(E) Exists and equals 1/2.

(F) None of the above.

Answer to 6:

(7) Suppose that the temperature at point (x, y) in the x-y plane is given by $f(x, y) = x+2y+3xy$. Which of the following vectors points in the direction one should move from the point $(x_0, y_0) = (2, 1)$ in order to <u>increase</u> temperature most rapidly?

(A) $(2, -1)$

(B) $(-4, -8)$

(C) $(-1, 2)$

(D) $(4, 8)$

(E) none of the above

Answer to 7:

(8) Find the most general solution to the differential equation $xy' - y = x^2 - x$.

(A) $y(x) = x - \ln x + C$

(B) $y(x) = x^2 - x\ln x + C$

(C) $y(x) = x^2 - x\ln x + Cx + D$

(D) $y(x) = x^2 - x\ln x + Cx$

(E) $y(x) = x^2 + x\ln x + Cx$

(F) none of the above

Answer to 8:

(9) The rate at which a student progresses through a take-home final is proportional to the number of days that have passed since the final was handed out. If the student has finished one eighth of the exam after four days, after how many days (counting from the day on which the exam was handed out) will the student have finished one half of the exam?

(A) 5 days (B) 6 days (C) 8 days

(D) 10 days (E) 12 days (F) none of the above

Answer to 9:

(10) What is the coefficient of x^4 in the power series solution of the differential equation $y'' - x^2y' = xy$ subject to the initial conditions $y(0) = 1$ and $y'(0) = 0$?

(A) 1

(B) -1

(C) 0

(D) $\frac{1}{6}$

(E) $\frac{1}{12}$

(F) none of the above

Answer to 10:

(11) If $y'' = 4y' - 4y + e^x$ and $y(0) = y'(0) = 1$, find $y(1)$.

(A) $3e^2 - e$ (B) $e^2 + 2e - 2$ (C) e

(D) $-e^2$ (E) $2e^2 - 2e + 1$ (F) none of the above

Answer to 11:

(12) Find the minimum and maximum values of the function $x + 2y - 3z$ on the surface $x^2 + 2y^2 + 6z^2 = 2$.

(A) 0 and 3 (B) -1 and 2 (C) -1 and 1

(D) $-\sqrt{3}$ and $\sqrt{3}$ (E) -3 and 3 (F) none of the above

Answer to 12:

(13) Evaluate the following integral by reversing the order of integration:

$$\int_0^8 \int_{x^{1/3}}^2 \frac{4}{y^4+1} dydx.$$

(A) $\ln 17$ (B) $\ln 15$ (C) $\ln 9$

(D) $\ln 8$ (E) $\ln 5$ (F) none of the above

Answer to 13:

(14) Let D be the region inside the circle $x^2 + y^2 = 1$ which lies above the x-axis and between the lines $y = x$ and $y = -x$. Using polar coordinates, compute the integral:

$$\iint_D y \, dA.$$

(A) $\pi/6$ (B) $\pi/12$ (C) $1/6$

(D) $1/3$ (E) $\sqrt{2}/3$ (F) none of the above

Answer to 14:

(15) The cup of a wine glass has the shape of the region inside the paraboloid $z = 2(x^2 + y^2)$ and under the plane $z = 2$. What is the volume of the glass?

(A) $5\pi/2$ (B) $7\pi/2$ (C) $\pi/2$

(D) π (E) $3\pi/2$ (F) none of the above

Answer to 15:

(16) Consider the cone $z = \sqrt{x^2 + y^2}$. Find the **surface area** of the part of the cone which lies between the planes $z = 1$ and $z = 2$.

(A) π
(B) 2π
(C) 3π
(D) 4π
(E) $3\pi\sqrt{2}$
(F) none of the above

Answer to 16:

(17) Let D be the region inside the sphere $x^2 + y^2 + z^2 = 4$, located above the plane $z = 0$ and under the cone $z = \sqrt{x^2 + y^2}$. Which of the following integrals expresses the volume of D?

(A) $\displaystyle\int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^2 \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$

(B) $\displaystyle\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$

(C) $\displaystyle\int_0^{2\pi} \int_0^1 \int_0^r r \; dz \; dr \; d\theta$

(D) $\displaystyle\int_0^{2\pi} \int_0^2 \int_0^r r \; dz \; dr \; d\theta$

(E) $\displaystyle\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} 1 \; dz \; dy \; dx$

(F) none of the above

Answer to 17:

(18) The function $f(x, y) = 3x^2 + y^2 + 4xy - 2x$ has:

(A) one critical point: a local max
(B) one critical point: a local min
(C) one critical point: a saddle point
(D) two critical points: a local max and a local min
(E) two critical points: a local min and a saddle point
(F) none of the above

Answer to 18:

(19) Find the equation of the tangent plane to the surface $x - z^3 = 1$ at the point $(0, 1, -1)$.

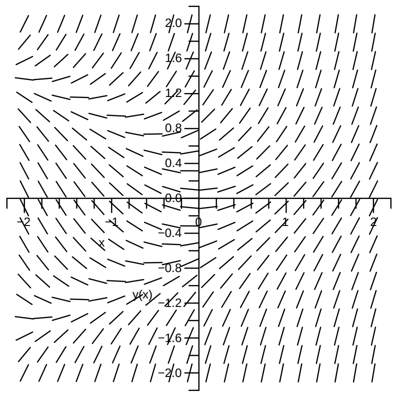
(A) $x - 3z - 3 = 0$ (B) $x + 3z + 3 = 0$ (C) $y - 3z - 4 = 0$

(D) $x - 3y - 3 = 0$ (E) $x + y - 3z = 4$ (F) none of the above

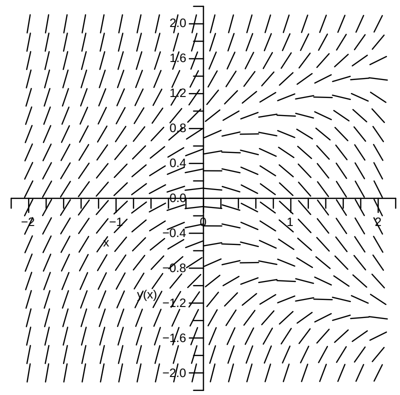
Answer to 19:

(20) Match up the differential equations with the direction fields.

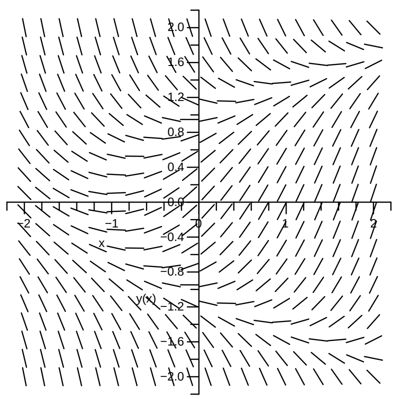
(A)



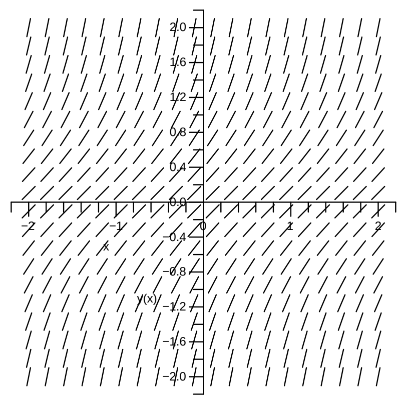
(B)



(C)



(D)



(i) $y' = y^2 + 1$

(ii) $y' = y^2 + x$

(iii) $y' = y^2 - x$

(iv) $y' = -y^2 + x + 1$

Answer to 20: (i) (ii) (iii) (iv)