MATH 114 – Sample Final Exam 2

1. Consider the line that is tangent to the cycloid given by the parametric equations $x = \theta - \sin\theta$, $y = 1 - \cos\theta$, at $\theta = \pi$. At what point $(0, y)$ does this line meet the y-axis?

   (a) (0,1) (b) $(0, \sqrt{2} - 1)$ (c) $(0, 0)$ (d) $(0, 2)$
   (e) $(0, \frac{\pi}{2} - 1)$ (f) $(0, 2 - \frac{\pi}{2})$ (g) $(0, \pi)$ (h) $(0, \pi - 2)$

2. The arc length of the curve given parametrically by $x = 3t^2$, $y = 2t^3$ for $0 \leq t \leq 1$ is

   (a) $3\pi/4 - 1/2$ (b) $\pi/2$ (c) 1 (d) $4\sqrt{2} - 2$
   (e) $6\sqrt{2} - 2$ (f) $6(\sqrt{2} - 1)$ (g) $\sqrt{10}$ (h) $3\ln 4$

3. What is the area inside one petal of $r = 2\sin(3\theta)$?

   (a) $\pi/3$ (b) $\pi/4$ (c) $2\pi^2$ (d) $\sqrt{2}\pi$
   (e) 4π (f) π (g) 2 (h) $3\pi/2$

4. Consider the graph given parametrically by

$$x = 1 + 2e^{\int_0^t \cos(\sqrt{s+1})\, ds} \qquad y = -3 - 6e^{\int_0^t \cos(\sqrt{s+1})\, ds}.$$

   Find the area between the graph and the x axis, and between the lines $x = -2$ and $x = 0$.

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5 (f) 6 (g) 7 (h) 8

5. The vector $\mathbf{V} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ is tangent to the plane $-3x + 2y + z = 20$. Find a vector that is tangent to this plane and perpendicular to $\mathbf{V}$.

   (a) $-3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (b) $\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ (c) $-\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ (d) $-\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$
   (e) $-3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ (f) $-3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ (g) $-2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ (h) $2\mathbf{i} - \mathbf{j} - \mathbf{k}$

6. The graphs of the equations $y = x^2$ and $y = x^3$ cross at the point (1,1). What is the cosine of the angle at which they cross (i.e., what is the cosine of the angle made by their tangent vectors at that point)?

   (a) 0 (b) $\frac{\sqrt{7}}{6}$ (c) $\frac{3\sqrt{2}}{5}$ (d) $\frac{2\sqrt{3}}{15}$ (e) $\frac{5\sqrt{3}}{16}$ (f) $\frac{3\sqrt{5}}{16}$ (g) $\frac{7\sqrt{3}}{20}$ (h) $\frac{7\sqrt{2}}{10}$

7. Consider the set of points given in cylindrical coordinates by $r = 4\cos\theta$. What 3-dimensional figure is given by these points?

   (a) A circle of radius 2 centered at (2,0).
   (b) A circular cylinder parallel to the z-axis centered at (0,0).
   (c) A cone centered at the origin opening along the z axis.
   (d) An elliptic paraboloid opening along the z-axis.
   (e) A sphere of radius 2 centered at the origin.
   (f) A circular cylinder parallel to the z-axis centered at (2,0).
   (g) A sphere of radius 2 centered at (2,0).
   (h) A hyperbolic paraboloid.

8. Consider the tangent plane to the surface $z = \sqrt{8 - 3x^2 - y^2}$ at the point (1,1,2). The part of this plane in the first octant, together with the three coordinate planes, bounds a tetrahedron. What is the volume of this tetrahedron?

   (a) 8 (b) $\frac{128}{9}$ (c) $\frac{256}{3}$ (d) $\frac{32}{3}$ (e) $\frac{64}{9}$ (f) $\frac{128}{3}$ (g) $\frac{32}{9}$ (h) $\frac{256}{9}$

9. A vector parallel to the direction of fastest increase of $w = 3x^2 - xy + z$ starting from the point $(1, -1, 6)$ is

   (a) $7\mathbf{i} - \mathbf{j} + 6\mathbf{k}$ (b) $12\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}$ (c) $7\mathbf{i} - \mathbf{j} + \mathbf{k}$ (d) $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$
   (e) $6\mathbf{i} - \mathbf{j} + \mathbf{k}$ (f) $12\mathbf{i} - \mathbf{j} - 6\mathbf{k}$ (g) $6\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ (h) $6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}$

10. The function $f(x, y) = -2x^3 + 4x^2 + 4y^2 + 4xy$ has

    (a) A local minimum and a saddle point.
    (b) A local maximum and a saddle point.
    (c) A local minimum and a local maximum.
    (d) Two local minima.
    (e) Two local maxima.
    (f) Two saddle points.
    (g) A local minimum and no other critical points.
    (h) A local maximum and no other critical points.

11. Find the *absolute* maximum and absolute minimum values of the function $f(x, y) = x^2 + 3y^2 + 2y$ on the set $\{(x, y)|x^2 + y^2 \leq 1\}$.

(a) max: 5, min: 1 (b) max: 5, min: 1/2 (c) max: 5, min: $-1/3$
(d) max: 5, min: $-1/2$ (e) max: 6, min: 1 (f) max: 6, min 1/2
(g) max: 6, min: $-1/3$ (h) max: 6, min: $-1/2$

12. Compute the double integral $\iint_T e^{x+y}\, dA$ over the triangle T that has vertices (0,0), (0,2) and (2,0).

(a) $1 + e^2$ (b) $e^2 - 1$ (c) e^2 (d) 1
(e) $2 + e^2$ (f) $e^2 - 2$ (g) $2e^2$ (h) $2e^2 - 1$

13. Use polar coordinates to evaluate

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1 + x^2 + y^2}\, dxdy.$$

(a) $\frac{\pi}{4} \ln 4$ (b) $\frac{\pi}{2} \ln 4$ (c) $\frac{\pi}{8} \ln 4$ (d) $\frac{\pi}{3} \ln 4$
(e) $\frac{\pi}{4} \ln 5$ (f) $\frac{\pi}{2} \ln 5$ (g) $\frac{\pi}{8} \ln 5$ (h) $\frac{\pi}{3} \ln 5$

14. Evaluate $\int_0^1 \int_{y^2}^1 ye^{x^2}\, dxdy$.

(a) $e - 1$ (b) $\frac{1}{2}(e - 1)$ (c) $\frac{1}{4}(e - 1)$ (d) $3(e - 1)$
(e) $e + 1$ (f) $\frac{1}{2}(e + 1)$ (g) $\frac{1}{4}(e + 1)$ (h) $3(e + 1)$

15. Let H be the top half of the solid ball of radius 1, centered at the origin. Tha is, $H = \{(x, y, z)|x^2 + y^2 + z^z \leq 1,\ z \geq 0\}$. Calculate

$$\iiint_H z^2\, dV.$$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{5}$ (e) $\frac{\pi}{6}$ (f) $\frac{2\pi}{3}$ (g) $\frac{2\pi}{5}$ (h) $\frac{2\pi}{15}$

16. Suppose $y = f(x)$ satisfies the second-order differential equation $y'' + 2y' + 2y = 0$ and $y(0) = 0$ and $y'(0) = 1$. Then $y(1) =$

(a) $\frac{\sin 1}{e}$ (b) $\frac{\cos 2}{e}$ (c) $\frac{\sin 2}{2}$ (d) $e^{-2}(\sin 1 + \cos 1)$
(e) 0 (f) $e^{-\frac{1}{2}}\left(\sin \frac{1}{\sqrt{2}} + \cos \frac{1}{\sqrt{2}}\right)$ (g) $\frac{1}{2}(e^2 + e^{-2})$ (h) -1

17. The family of curves $y = \frac{x}{2} + \frac{C}{x}$ are all solutions of which of the following differential equations?

(a) $y' + y = 1$ (b) $y' - y = x$ (c) $y' + \frac{y}{x} = 1$ (d) $y' - \frac{y}{x} = 1$

(e) $y' + \frac{y}{x} = \frac{1}{x}$ (f) $y' - \frac{y}{x} = \frac{1}{s}$ (g) $\frac{yy'}{x} = 1$ (h) $\frac{xy'}{y} = 1$

18. The amount y of a certain substance varies according to the "logistic equation" $\frac{dy}{dt} = 10y - y^2$. If $y(0) = 2$, then $\lim_{t\to\infty} y(t) =$

(a) 0 (b) 2 (c) 5 (d) 10 (e) e^10 (f) e^2 (g) e^5 (h) ∞