MATH 114 – Sample Final Exam 1

1. A curve is given in parametric form by $x = e^{t^2-1}$, $y = t^3 + 1$. Its tangent line at the point (1,0) is

   (a) $y = -\frac{3}{2}x + \frac{3}{2}$ (b) $y = -x + 1$ (c) $y = \frac{2}{3}x - \frac{2}{3}$ (d) $y = x - 1$ (e) $y = 0$

2. The arc length of the curve given in polar coordinates by $r = e^{\theta}$ for $0 \leq \theta \leq 3\pi$ is

   (a) $2e^{6\pi}$ (b) $\sqrt{2}e^{3\pi}$ (c) $2e^{3\pi}$ (d) $\sqrt{2}(e^{6\pi} - 1)$ (e) $\sqrt{2}(e^{3\pi} - 1)$

3. The area of the region *inside* the cardioid $r = 2(1 + \sin\theta)$, *outside* the circle $r = 2\sin\theta$ and *above* the x-axis is

   (a) $4 + \pi$ (b) $1 + 2\pi$ (c) 3π (d) $8 + 2\pi$ (e) $2 + 2\pi$

4. Consider the graph given parametrically by $x = t^3 + 1$, $y = 1 - t^2$. Find the area under the graph, over the x axis, and between the lines $x = 1$ and $x = 2$.

   (a) 1/3 (b) 2/5 (c) $\sqrt{2}/7$ (d) 13/17 (e) $3\pi/8$

5. What is the equation of the plane that is perpendicular to the vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and that passes through the point (3,2,1)?

   (a) $3x + 2y + z = 14$ (b) $x - y + 2z = 3$ (c) $x + y + z = 6$
   (d) $3x + 2y + z = 3$ (e) $x - y + 2z = 6$

6. What is the angle between the vectors $2\mathbf{i}$ and $5\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$?

   (a) 0 (b) $\pi/8$ (c) $\pi/6$ (d) $\pi/4$ (e) $\pi/3$

7. The four vertices of a regular tetrahedron are $V_1 = (1, 0, 0)$, $V_2 = (-1/2, \sqrt{3}/2, 0)$, $V_3 = (-1/2, -\sqrt{3}/2, 0)$ and $V_4 = (0, 0, \sqrt{2})$. What is the cosine of the dihedral angle between any pair of faces of the tetrahedron? (The dihedral angle is the angle between the planes containing the faces).

   (a) 0 (b) 1/3 (c) 1/2 (d) 2/3 (e) 1

8. Consider the tangent plane to the graph of $z = x^2y - y + e^x + 1$ at (0,1,1). This plane meets the x-axis at the point where $x =$

   (a) -2 (b) 0 (c) 1 (d) 3 (e) 6

9. Suppose $z = x^2y + y^2$, where x and y are each functions of t. When $t = 0$, we are given that $x = 1$, $y = 2$, $dx/dt = 3$ and $dy/dt = 4$. What is dz/dt when $t = 0$?

   (a) 0 (b) 11 (c) 17 (d) 27 (e) 32

10. The function $f(x,y) = x^3 + 3x^2 - y^2$ has

(a) two local maxima, no local minima, and no saddle points.
(b) no local maxima, two local minima, and no saddle points.
(c) no local maxima, no local minima, and two saddle points.
(d) no local maxima, one local minimum, and one saddle point.
(e) one local maximum, no local minima, and one saddle point.

11. The product of the maximum and minimum values of the function $f(x,y) = xy$ on the ellipse $\frac{x^2}{9} + y^2 = 2$ is

(a) 12 (b) -12 (c) 9 (d) -9 (e) 0

12. Let S be the square in the xy-plane with vertices (0,2), (0,3), (1,2), and (1,3). Find the volume of the solid region lying over the square S and under the graph of $z = ye^{xy}$.

(a) 1 (b) 3π (c) $e^2 + 6$ (d) $2\pi e^3$ (e) $e^3 - e^2 - 1$

13. Let R be the region in the plane lying in the first quadrant, below the graph of $y = x^2$ and to the left of the line $x = 1$. Evaluate $\iint_R 2x \cos y \, dA$.

(a) 0 (b) $1 + \sqrt{2}$ (c) $2\pi/3$ (d) $17/6$ (e) $1 - \cos 1$.

14. Evaluate $\int_0^{\sqrt{\frac{\pi}{2}}} \int_y^{\sqrt{\frac{\pi}{2}}} \cos(x^2) \, dxdy$.

(a) 1/2 (b) 1 (c) π (d) $\sqrt{\pi/2}$ (e) 2

15. Let D be the region $D = \{(x,y) | x^2 + y^2 \leq 1, \ x \geq 0, \ y \geq 0\}$. Evaluate $\iint_D e^{-x^2-y^2} \, dA$.

(a) $1 + 1/e$ (b) $\pi(1-e)$ (c) $(\pi/4)(1 - 1/e)$ (d) $1 - 1/e$ (e) π

16. Let B be the region (solid ball) bounded by the unit sphere $x^2 + y^2 + z^2 = 1$. Compute

$$\iiint_D \exp\left((x^2 + y^2 + z^2)^{\frac{3}{2}}\right) \, dV.$$

(Here "exp" is the usual exponential function, i.e., $\exp(x) = e^x$).

(a) $\frac{4\pi}{3}(e^{5/2} - 1)$ (b) $\frac{4\pi}{3}(e - 1)$ (c) $\frac{\pi}{3}(4e^{3/2} - 1)$ (d) $\frac{\pi}{3}(8e^{1/2} - 4)$ (e) $\frac{2\pi}{3}(e^{5/2} - e)$

17. Find a function $u(x,t)$ with the properties that $\frac{\partial u}{\partial t} = 2u$, while on the line $t = 0$ we have $u(x,0) = 3x + 7$.

(a) $e^{2t} + 3x + 6$ (b) $e^{-2t} + 3x + 6$ (c) $(3x + 7)e^{2t}$ (d) $(3x + 7)e^{t+2}$ (e) $3x + 7e^{2t}$

18. A particle moves along the y-axis in such a way that $tv + y = 4t^3$, where y is the position of the particle at time t and v is the velocity of the particle at time t. At time $t = 1$, the particle is at the point $y = 2$. What is the position of the particle at time $t = 2$?

(a) $y = 1$ (b) $y = 17/2$ (c) $y = 26/3$ (d) $y = 28\sqrt{2} - 1$ (e) $y = \ln 4$

19. Let $y = f(x)$ be a function such that $y'' - 2y' + 2y = 0$. Suppose that the line $y = 1$ is tangent to the graph of $y = f(x)$ at $x = 0$. Then $f(x) =$

(a) $2e^x \cos x + e^x \sin x$ (b) $e^{2x} - 2e^x$ (c) $\cos 2x + \sin 2x$
(d) $e^x(\cos x - \sin x)$ (e) $e^x \cos 2x + e^x \sin x$

20. A certain function $y = f(x)$ satisfies the differential equation $y'' = y + 2x$, and the graph of $y = f(x)$ passes through the origin. Also, $f'(0) = 2$. What is $f''(1)$?

(a) $2\pi e$ (b) $\cos 1$ (c) $e^2 + e^{-2} + 2$
(d) $3e + 2e^{-1} - 2$ (e) $2(e - e^{-1})$